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Accelerated Learning Programme

ALP MATHEMATICS

Glossary for all Units,
and the full text
of the Unit "Basic Geometry Concepts",
translated in English

Funded by the Asylum, Migration and Integration Fund of the European Union

IOM UN MIGRATION

ΠΑΝΕΠΙΣΤΗΜΙΟ ΘΕΣΣΑΛΙΑΣ

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ΠΑΝΕΠΙΣΤΗΜΙΟ ΘΕΣΣΑΛΙΑΣ

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ΝΤΟΡΕΤΤΑ ΑΣΤΕΡΗ

TABLE OF CONTENTS

BASIC GEOMETRY CONCEPTS

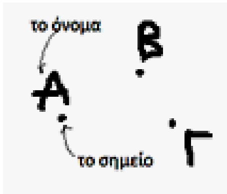
A.1 – Lines

Point: we can understand what a point is, by thinking of the intersection between two lines, a pencil mark on a paper, or a full-stop.

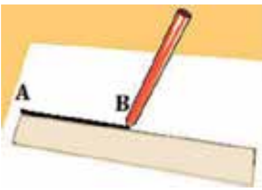
+	,	-	.	/
;	<	=	>	?

Points have no **length** or **width**

We sometimes give name to points in geometry. These names are **capitals letter**, e.g., **A, B, C.**



(picture on the right: το όνομα: name
το σημείο: point)



The shortest path between two points (e.g., A and B) is a ***line segment*** (a thread connecting A and B). We draw line segments using a ruler. We give a name to line segments according to its start and end point. This means that we can talk about ***"line segment AB".*** The order of the letters is not important. 'Line segment AB' is the same as 'line segment BA'.

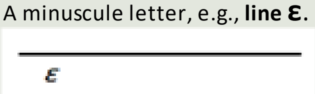
Line segments only have length; they have no width

Lines

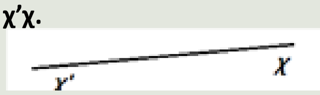
Line: We can understand what a line is by thinking of a tightrope. We can draw a line by using a ruler to indefinitely extend a line segment AB to the left of A and to the right of B. Lines do not have a limit (a start or end point).

A line can be named using

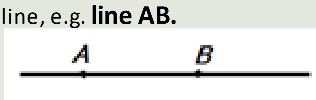
A minuscule letter, e.g., **line ε.**



Two identical miniscule letters (one of them accented) e.g **line χ'χ.**



The name of two points on the line, e.g. **line AB.**



Activity A.1.2 (discuss in your group)

Imagine a point O.

- Draw a line that passes through O.
- Can you draw another one?
- How many lines do you think can pass through O?

Now imagine two points, A and B.

- Draw a line that passing through both A and B.
- Can you draw another one?
- How many lines do you think can pass through both these points?

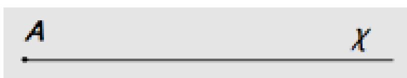
Complete & Learn

There .... passing through two points.

There .... passing through a single point.

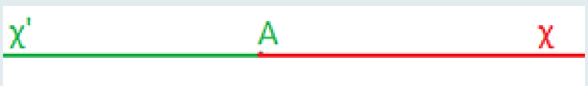
Half lines

Extend a line segment AB only to one direction. The new shape is a **half line.** A half line has a beginning but no end. **The name of a half line** is: a capital letter (its beginning) and a small letter (one of the final letters in the alphabet, e.g., *x, y*).



For example, we might talk about **half line Ax.** The letter *x* does not represent any points.

A point A on a line χ'χ defines two half lines, Aχ' and Aχ.



Activity A1.3

A. a) How many line segments do the **points *A, B*** and ***C define in each case?***

b) How many lines do the points *A, B* and *C define in each case?*

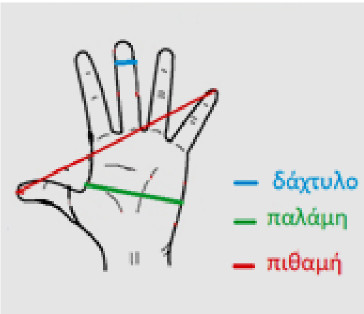
B. Draw ***four points A, B**, **C*** and ***D***. In how many different ways can you do this? (depending on how many points are on each line)

Measuring length

Measuring length, or distance, was one of the first things humans needed to do.

measuring = comparing

Measuring a length means ***comparing it with another length that is known and unchanging*** and estimating how many times **it is larger or smaller than it**. This known, unchanging length is called a **unit of measurement**.



In the distant past, people used body parts to measure length. They used fingers, handbreadths, or spans. (One handbreadth □ 4 fingers, one span □ 2 handbreadths)

cubit

foot

step

- δάχτυλο: finger
- παλάμη: handbreath
- πιθαμή: span

For lengths that were somewhat larger, they also used part of the arm, from the elbow to the fingertips. This was called a **cubit** (1 cubit □ 4 handbreadths). They also used **feet** and **steps.**

Activity (Groupwork)

A) Measure the length of your desk, using your handbreadth as a unit of measurement. How many handbreadths long is it? How many fingers does that equal?

B) Measure the width of your notebook, using your finger as a unit of measurement. How many fingers wide is it? How many handbreadths does that equal?

Γ) Measure the width of your classroom, using a foot as a unit of measurement. How many feet wide is it How many handbreadths does that equal?

Compare your results. How different are they? Why is that?

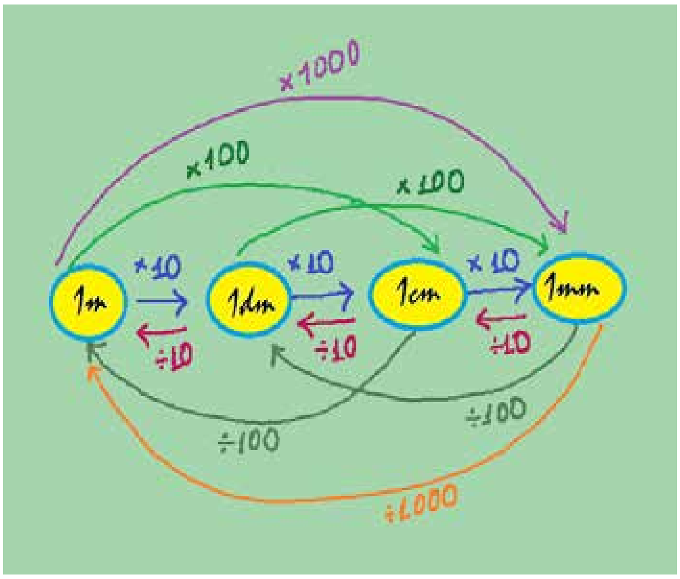
Measurements with these units were not always the same. For example, every person has a different a handbreadth. That’s why most countries decided to use **the same unit for measuring length** in France in 1791. They decided to use $\frac{1}{10.000.000}$ of the distance between the Equator and the North Pole as a unit of measurement. They called this distance a **meter.**

Every unit of measurement can be divided into smaller ones. So, we can divide 1 meter into 10 parts (dm), each one of them into another 10 (cm) and each one of them into another 10 (mm).

dm = decimeter = tenth.
cm = centimeter = hundredth.
mm = millimeter = thousandth.

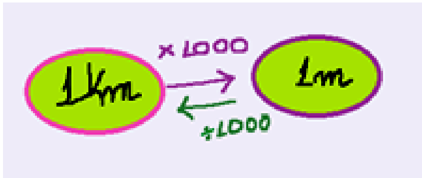
1m = 10 dm = 100cm = 1000mm
1dm = 10cm = 100 mm
1cm = 10mm

For larger distances, we use a **kilometer (Km)** as a unit of measurement. 1km = 1000m



These shapes show us how to translate one unit into to another. For example:

3cm = (3 x 10) = 30mm
12dm = (12 x 100) = 1200mm
3,2m = (3,2 x 100) = 320cm
2,1km = (2,1 x 1000) = 2100m
200cm = (200 :100) = 2m
120mm = (120 : 100) = 1,2dm



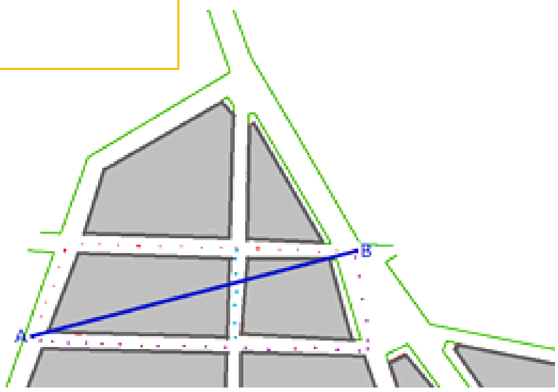
Exercise A.1.1

Complete the following equalities:

1. 320cm = ……………… dm = …………………m
2. 5,2 km = ……………………. m = ………………………… mm
3. 45000 mm = ……………… m
4. 20dm = …………….. mm
5. 780 cm = ……………m = ………………km

Exercise A.1.2

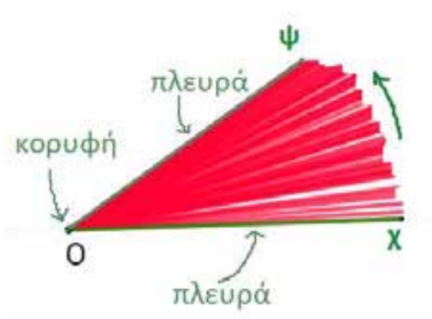
Find and draw the shortest distance from position A to position B (Gray areas are buildings, so you can't walk there).

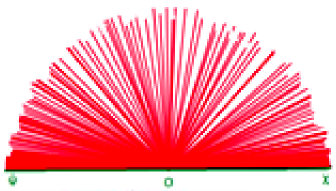


A.2 – Angles

Turn – rotate – revolve around…

Imagine two half lines O*x*, O*y* with a common beginning, O. Rotate ***Oy around O** in the direction that you see*. Then ***Oy*** covers ("paints") *the red area*. This area, including the half lines O*x*, O*y* and point O is an **angle**. The half lines O*x*, and O*y* are the **sides of the angle**; point O is the **apex of the angle**. We can extend the sides of the angle as far as we want.

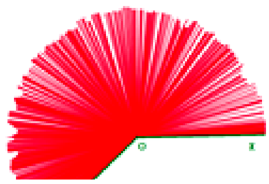




When a half line O*y* and the half line O*x* form a line, this is a **straight angle.** Any angle before that point is a **convex angle.** If O*y* continues to turn, then the angle is a **reflex angle**.

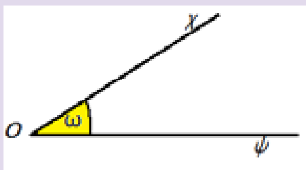
When O*y* makes a full turn and covers O*x*, then the angle is a **full angle**.

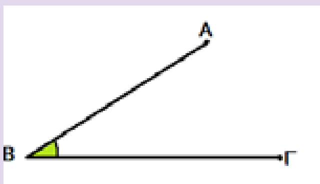
Before O*y* starts rotating, it covers O*x*. Then the angle is a **null angle**.



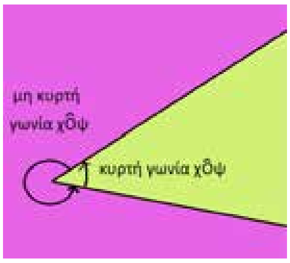
Naming angles

- ***A minuscule letter***, e.g., angle $\hat{\omega}$
- ***Or a capital letter*** (if no other angle in the figure shares the same apex) e.g., angle $\hat{B}$
- ***or 3 letters;*** The apex is always in the middle! e.g., angle $\widehat{AB\Gamma}$ ή $\widehat{\Gamma BA}$ ή angle $\widehat{\chi O\psi}$ ή $\widehat{\psi O\chi}$





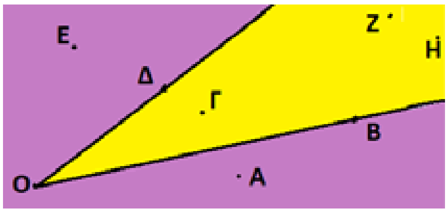
Two half lines O*x* and O*y* with the same apex O always form 2 angles, one of which is convex and the other reflex.



When we talk about angle $\widehat{xOy}$, we mean the convex angle. If we want to talk about to the reflex angle, we say: ***"the reflex angle** $\widehat{xOy}$*».

reflex angle xOy

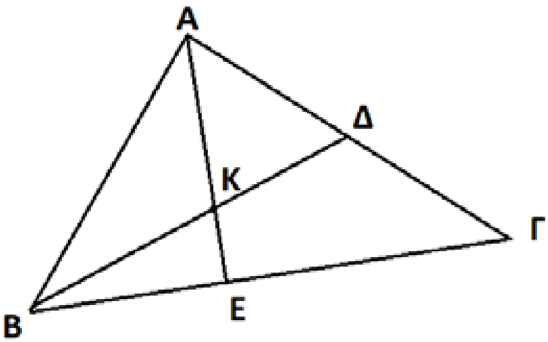
convex angle xOy



Angles have: **an apex, sides,** and **internal points.** Which of the points you see in the figure on the right <u>are parts of the angle $\hat{O}$</u>?

………

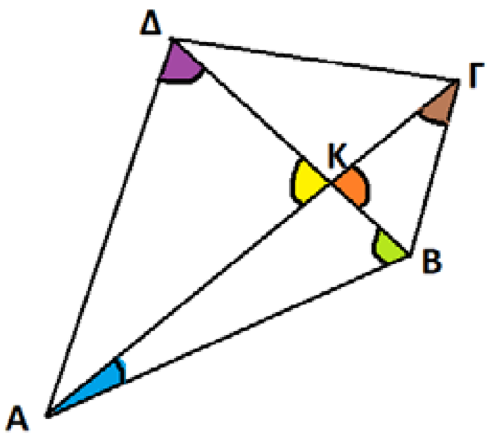
<u>Exercise A.2.1</u>: Let's draw!



Use ***blue*** to colour the angles:
a) $\widehat{BKE}$ b) $\widehat{AB\Delta}$ c) $\widehat{B\Delta\Gamma}$

Use ***red*** to colour the angles:
a) $\widehat{KBE}$ b) $\widehat{A\Delta B}$ c) $\widehat{KE\Gamma}$

<u>Exercise A.2.2</u>: Write the names of the angle.



Name the following angles using three letters:

a) **orange:** …………………………..

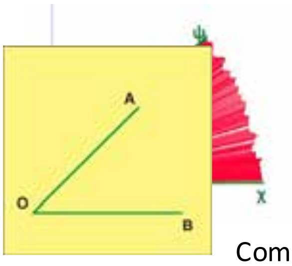
b) **green**: ………………………………..

c) **blue**: ………………………………….

d) **yellow**: ……………………………….

e) **brown**: ………………………………….

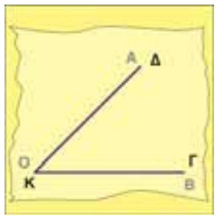
f) **purple**: ………………………………….



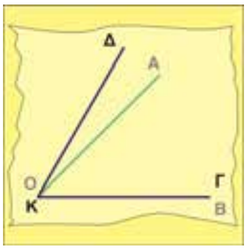
Activity A.2.1: Compare angles

We want to compare angle $\widehat{BOA}$ and angle $\widehat{\Gamma K\Delta}$. This is like transporting angle $\hat{K}$ onto $\hat{O}$, in such a way that the apexes O and K and their sides KΓ and OA *cover each other.* What will then happen to sides KΔ and OB?

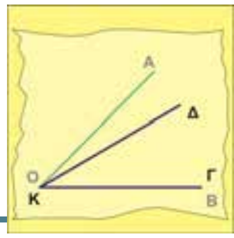
Complete the gaps with the suitable word *(smaller, greater, equal)* or the appropriate symbol *(<, >, =)*



a) Perhaps the other side will also be superimposed.[1] If that happens, then the angles are …………………. ; so we write that $\widehat{AOB}$ … $\widehat{\Gamma K\Delta}$

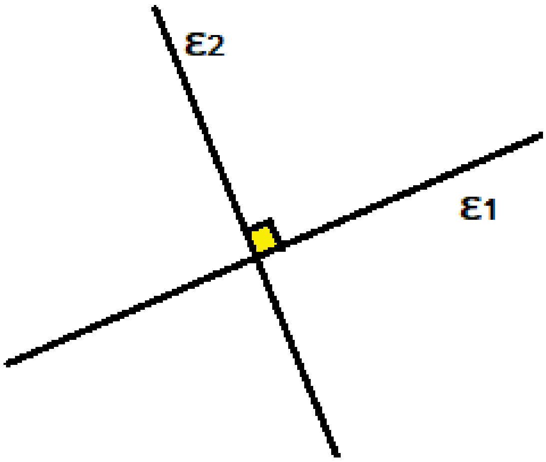


b) Or it will fall inside $\hat{O}$. This means that $\hat{K}$ is ………………………………. than $\hat{O}$ (not so wide); so we write that $\widehat{AOB}$ … $\widehat{\Gamma K\Delta}$



c) Or it will fall outside $\hat{O}$. This means that $\hat{K}$ is……………………. than $\hat{O}$ (wider), so we write that $\widehat{AOB}$ … $\widehat{\Gamma K\Delta}$

Two lines are **perpendicular** when they intersect in such a way that they form **four equal angles.** These angles are **right angles.**



My new words

………

………

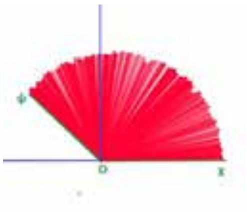
………

………

………

[1] This means that one falls onto the other and covers it.

Any angle that is smaller than a right angle is acute.



Any angle that is greater than a right angle but smaller than a straight one is **obtuse**.

Exercise A.2.3: Types of angles

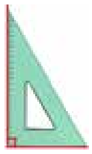
Draw an angle of each type.

Null angle	Acute angle
Right angle	Obtuse angle
Straight angle	Reflex angle
Full angle	

A.3 – Distances

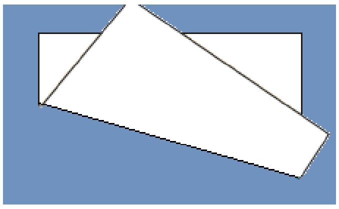
Perpendiculars from… to….

To properly draw a line that is perpendicular to another, you need to use a right triangle. This is a triangle with perpendicular sides that form a right angle.



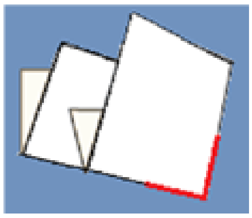
If you don't have a right triangle you can create one using <u>a sheet of paper</u>.

<u>Step 1</u>: Fold the paper once

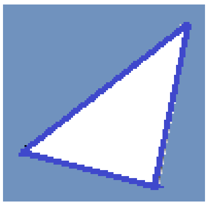


<u>Step 2</u>: Fold it again along the first fold. This angle is a perfect right angle.

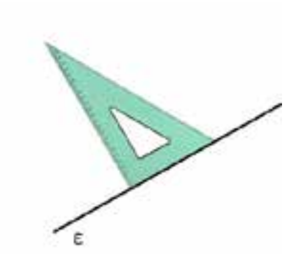
If you unfold the paper, this creates two perpendicular lines!



<u>Step 3</u>: If you like, you can also fold the rest of the paper so that it is shaped like a triangle.



Perpendicular from a point A from a line ε



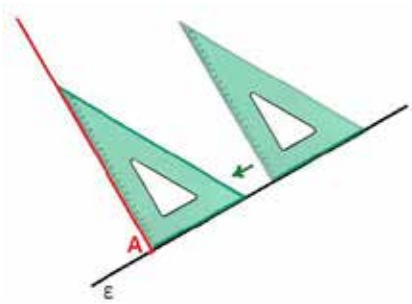
Imagine a line ε. You want to draw another line, ζ, which is perpendicular to ε.

Place your right triangle so that one of its right sides (the smaller of the two) sits on line ε.

But there's one more thing you need to know.

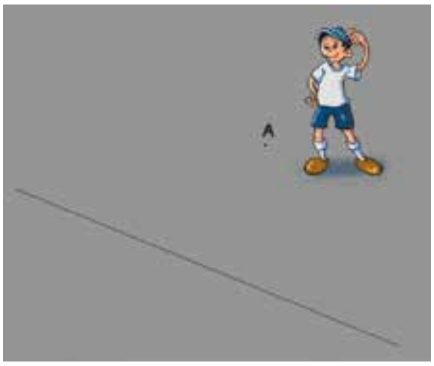
What point of line ε do I want the perpendicular line to pass through?

Let's assume that you want to make the lines intersect **on point A.** Slide the triangle along line ε, until its angle meets point A**. Then you can draw the perpendicular to ε from A.**



Activity A3.1

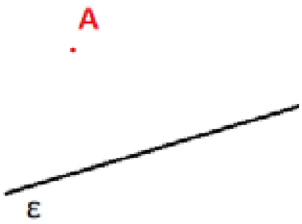
We often calculate the shortest distance to reach a point. Can you help this boy by drawing the shortest line to the sidewalk?



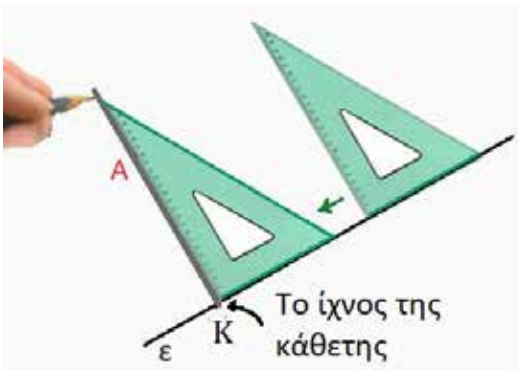
This problem is the same as finding **the shortest distance from a point to a line**. Discuss this with your group and find a solution.

Perpendicular from a point A outside line ε

Imagine a line ε and a point A outside it.



Place your triangle with one perpendicular side on the line ε. Slide it on the line. Stop when the other right angle meets point A. Draw the line using the other right side of the triangle. The perpendicular line intersects ε on K. **Point K** is the ***foot of the perpendicular***.

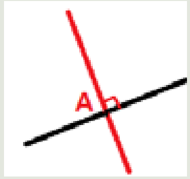


foot of the perpendicular

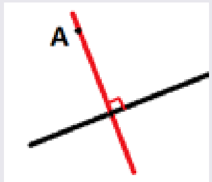
We can only draw one perpendicular to a line ε from a point outside it. The right line segment AK from point A to line ε is the shortest path to it. ***The length of the right line segment AK is the distance from the point to the line.***

A line cannot be perpendicular to itself. It is always perpendicular **to another line…**

… **perpendicular from** a point



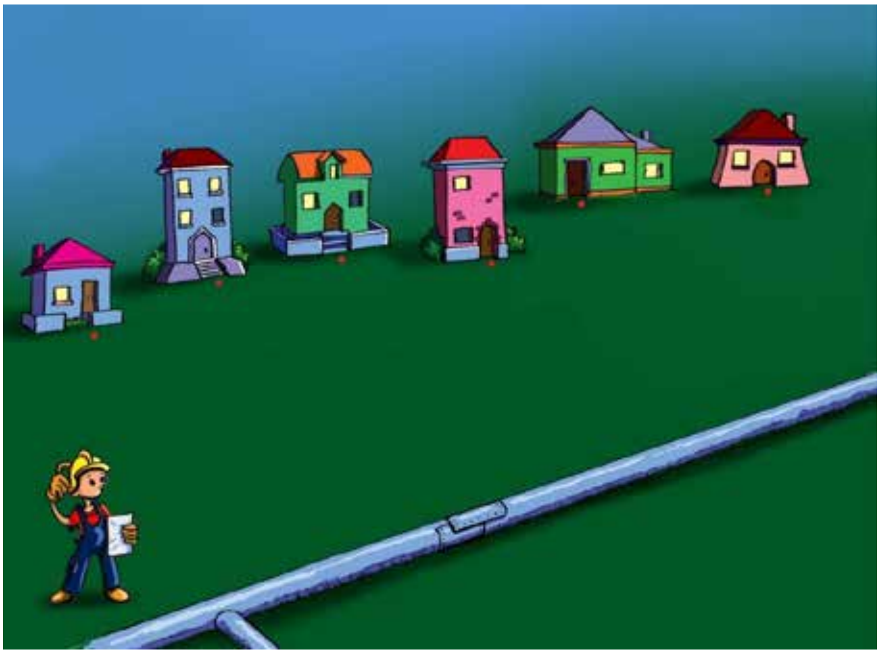
…or **perpendicular** from a point



Activity A3.2 (group work)

The houses in the picture are to be connected with the main water pipe. Every home owner will pay some money. The money will depend on the distance of the house from the water pipe. The cost is 57€/m (every 1cm in the picture represents 1m).

1. Which owner will pay the most money?
2. Which owner will pay the least money?
3. Will some owners pay the same amount of money?

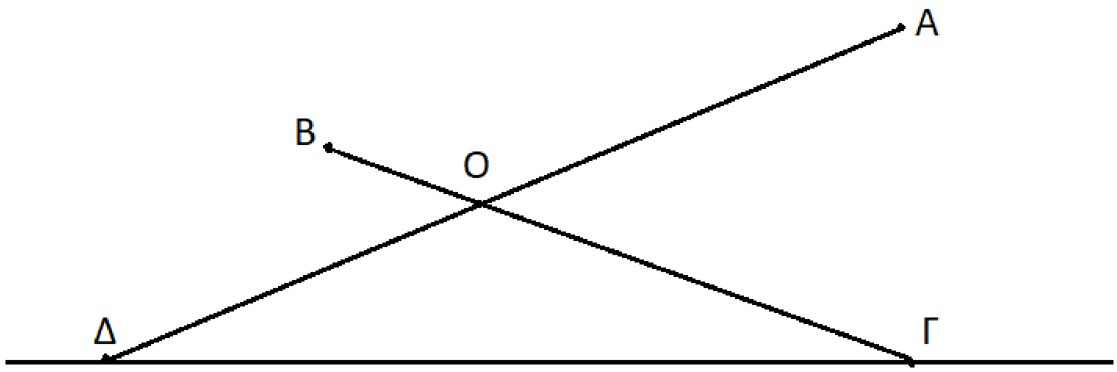


House	Distance from main water pipe	Connection costs
A		
B		
C		
D		
E		
F		

Practice

In the following ficure, you should draw:

i. A perpendicular **from O to line ΔΓ**
ii. A perpendicular **from A to line BΓ**
iii. A perpendicular **from A to line ΔΓ**
iv. A perpendicular **from B to line ΔA**
v. A perpendicular **from B to line ΔΓ**
vi. A perpendicular to BΓ **on** B
vii. A perpendicular to ΔA **on** A



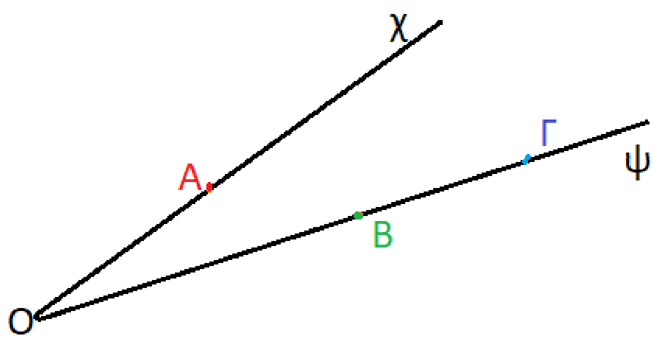
Perpendiculars from ... on...

Using a right triangle, draw:

A. A perpendicular **from point B to O*y*.**

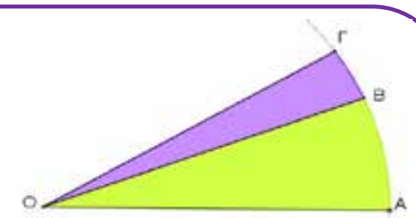
B. A perpendicular **from point B to O*x***.

C. A perpendicular to O*x* **on point Γ**.



A.4 – Adding and subtracting angles

In the figure on the right, what do angles $\widehat{AOB}$ and $\widehat{BO\Gamma}$ have in common?



....

....

We use the name ***adjacent*** for these angles.

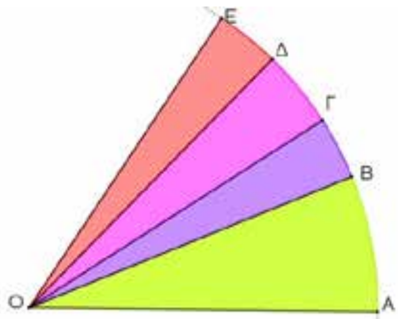
We can add such angles. Here's how:

$A\hat{O}B + B\hat{O}\Gamma = A\hat{O}\Gamma$

This also means that: $A\hat{O}\Gamma - A\hat{O}B = BO\Gamma$

All the points that make up angle $A\hat{O}B$ are also points of angle $A\hat{O}\Gamma$. $A\hat{O}B$ is also a part of angle $A\hat{O}\Gamma$.

Adding and subtracting angles



In this figure, the differently coloured angles are ***consecutive***.

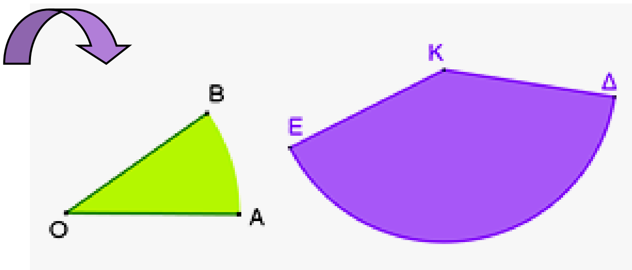
We can add consecutive angles.

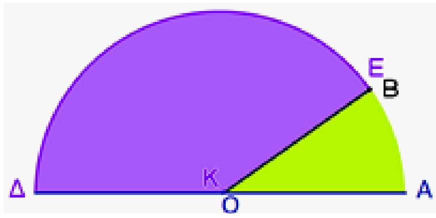
These additions also suggest the corresponding subtractions.

Complete the equalities:

1. $B\hat{O}\Gamma + \Gamma\hat{O}\Delta =$ ________
2. $B\hat{O}\Delta + \Delta\hat{O}E =$ ________
3. $B\hat{O}\Gamma + \Gamma\hat{O}E =$ ________
4. $B\hat{O}\Delta - B\hat{O}\Gamma =$ ________
5. $B\hat{O}E - B\hat{O}\Delta =$ ________
6. $B\hat{O}E - \Gamma\hat{O}E =$ ________

Two angles whose sum makes a straight angle are **supplementary**.

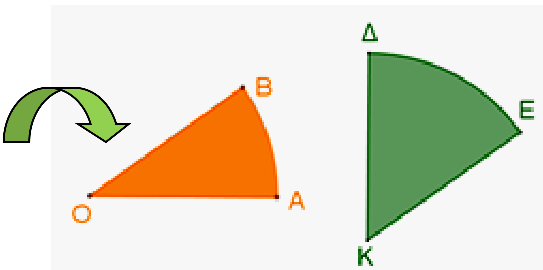


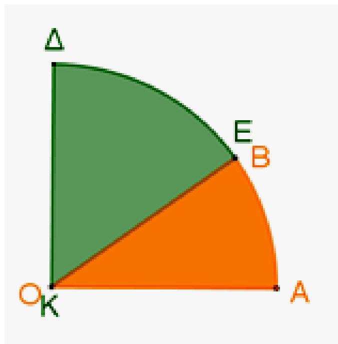


Supplementary angles

Adjacent & supplementary angles

Two angles whose sum is a right angle are **complementary**.





Complementary γωνίες

Adjacent & complementary angles

Exercise:

A) Draw i) an acute angle and ii) an obtuse angle and then design their supplementary angles.

B) Design an acute angle and its supplementary angle.

My new words

....

....

....

....

....

....

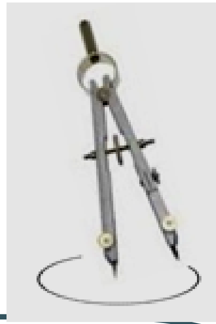
....

....

A.5 – Circles

Circles are a shape we often see around us.

How do we design a circle? We use a special instrument called a ***compass.*** *We hold one arm of the compass steady, and we rotate the other one until it has made a **full turn**.* In this way, we draw a line by keeping **a steady distance from a point (the centre).**



This means that a circle is a shape where all the points have the same distance from the point we call its centre.

Activity A5.1

Think and discuss how you might draw a circle without a compass

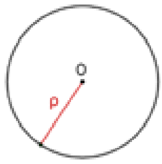
A) On soil or sand

B) On the whiteboard, using your hand

C) Using your body

D) On paper, using another object

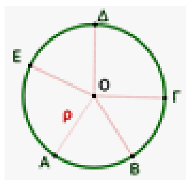
*The distance of every point in a circle from its centre is the **radius** of the circle. In Greek textbooks, we use the letter **ρ** to represent this (or **r** in other languages).*



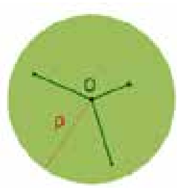
The notation ***circle (O, ρ)*** describes a circle where point O is the centre and ρ is the radius.

Circles with the same radius are equal.

A circle (O, ρ) divides the points that make up a plane into three groups:



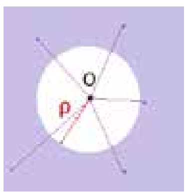
A) Any points in the plane whose distance from O is ***equal to ρ.*** These are the points of the circle.



B) Any points in the plane whose distance from O is ***smaller than ρ.***

The surface that contains all the points in groups (A) and (B) is a **disc.**

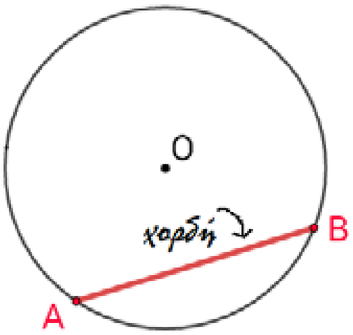
C) Any points in a plane whose distance from O is **greater than ρ.** These are the points ***external to the circle***.



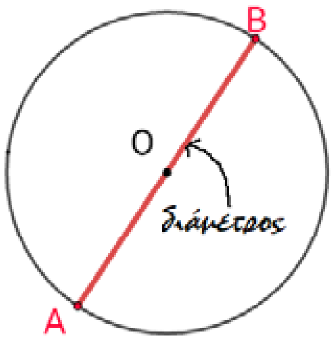
Activity A5.2

There is a mobile telephony mast in point K. The authorities want to build a school. To make sure that children are healthy, its best that the school is at a distance ***greater than 300m*** from the mast. To make sure that connections are good, it's best that the school is in a distance ***smaller than 1500m*** from the mast. Where can the school be built?

Let's learn some more things about circles



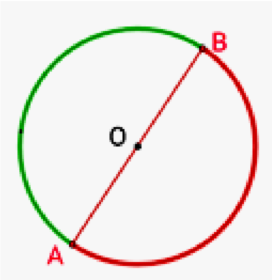
Το ευθύγραμμο τμήμα που ενώνει δύο σημεία πάνω στον κύκλο, το λέμε ***χορδή***



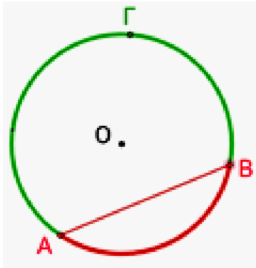
Τη χορδή που περνάει από το κέντρο του κύκλου, τη λέμε ***διάμετρο***

the line segment that connects two points on a circle is a *string*.

A string that passes through the centre of a circle is its *diameter*.

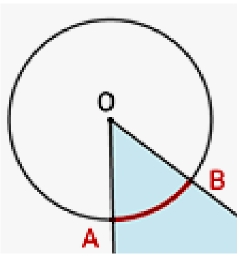


The end points A and B of the diameter AB divide the circle into <u>two equal parts</u>. Each one of them is called a **semi-circle**.[2]



Any two points in a circle divide it into *two parts*. Each one of these is called an **arc**. Arcs are represented as $\overset{\frown}{AB}$. We can add a third point to the notation, to show the largest arc, arc $\overset{\frown}{A\Gamma B}$.

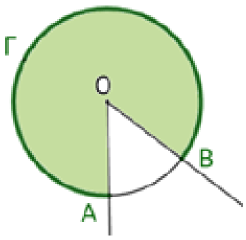
Central angles



Let's draw two half lines OA and OB, ***starting at the centre O*** of a circle. The angle that these create, $A\hat{O}B$, is a **central angle**.

The sides of the central angle $A\hat{O}B$ "cut" a piece of the circle: arc AB. Arc AB is the ***arc that corresponds*** to the central angle $A\hat{O}B$.

The same half lines also define the ***reflex angle*** $A\hat{O}B$. $A\Gamma B$ is the arc that corresponds to the reflex angle $A\hat{O}B$.



- ***There is only one central angle for each arc in a circle.***
- ***Each central angle in a circle has only one corresponding arc.***

A.6 – Measuring angles & arcs

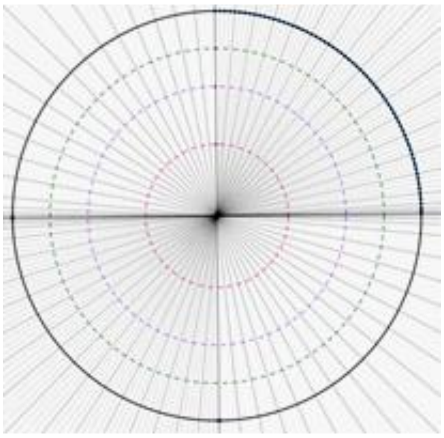
We need a unit of measurement to measure angles. This unit is a ***degree.*** We use ***the symbol*** ^{o} to represent degrees.

Let's divide a circle into 360 equal pieces. Each one of this pieces corresponds to just one central angle, which measures one degree (**1°**). This means that ***every circle measures 360°*** .

(Does the radius of the circle make a difference? Discuss this in class).

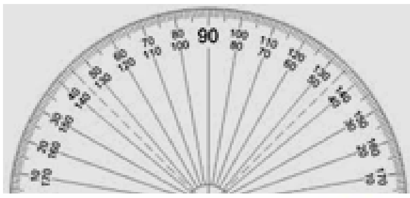
[2] Semi-circle: half circle.

Complete this table by writing down the measurement of each angle that is recorded in the left column.



Angle	Measuremet
Right angle	
Straight angle	
1/6 of a circle	
1/3 of a circle	
1/8 of a circle	
1/12 of a circle	

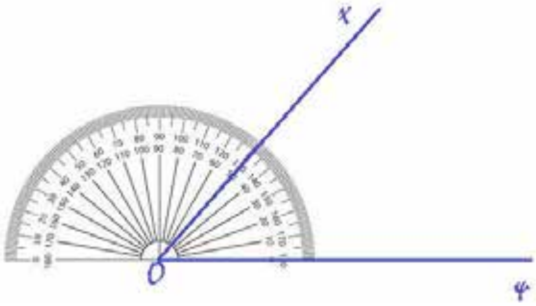
We use a ***protractor*** to measure angles. This instrument is a half-circle divided into degrees.



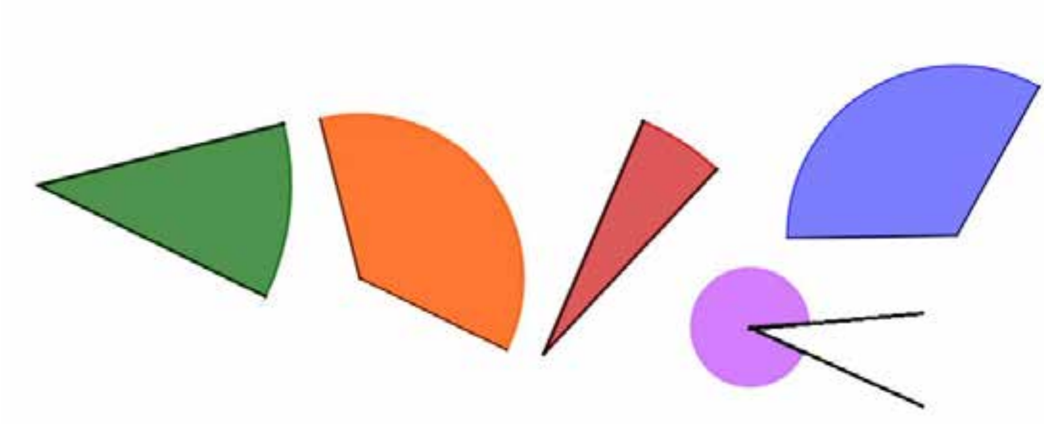
How do we measure angles?

We place the protractor on the degree in such a way that:

1. The centre of the protractor is placed on the apex of the angle.
2. One side of the protractor is on the line that passes through 0.
3. Starting from 0, we check the measurement that corresponds to the other side of the angle.



Exercise A.6.1: Measure the angles in the picture below

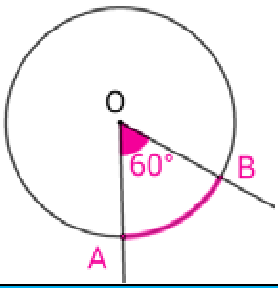


Μέτρηση τόξου

(How do we measure the arc of a circle?)

Every arc has just **one central angle**.

The measure of a central angle is the same as the measure of its corresponding ark.



In other words, arcs are also measured in degrees. For example, an arc that corresponds to a 60° angle, also measures 60° , so we note that $AB = 60^{o}$.

Activity A.6.1 (Discuss with your group)

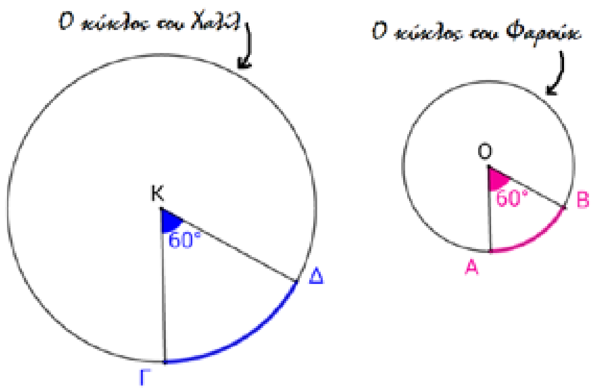
Halil (Χαλίλ) and Faruk (Φαρούκ) have drawn a circle and a 60° central angle. Here's what they say:

— Halil: *These arcs are equal!*

— Faruk: *No, they aren't! How can they be equal? Can't you see that arc ΓΔ is larger?*

— Halil: *But they are both 60° !*

What do you think? Who's right?



The measurement of an arc <u>shows</u> *what part (fraction) of a circle this is. That's why we can only compare arcs when they are on the same circle or on equal circles.*

A circle is a full angle. Full angles measure 360°. This means that ***a 60° angle is 1/6 of a circle***

$\left(\textit{think}: 6\times60° = 360° \text{ therefore } \frac{60^{o}}{360^{o}} = \frac{1}{6}\right)$.

Angle in degrees	Fraction of a circle
90°	
180°	
30°	
45°	
120°	
270°	

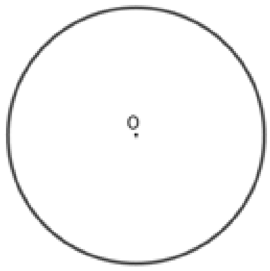
Complete the right column in this table. What part (fraction) of a circle does each angle on the left column correspond to?

Activity A.6.2

Divide this circle into 4 equal parts

A) with a protractor

B) without a protractor.



Exercises

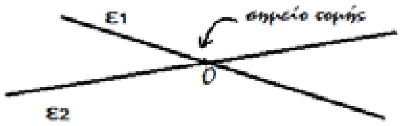
1. Draw a 35° angle. Next, draw its complementary angle.
2. Draw an angle that is $\frac{2}{3}$ of a right angle.
3. Draw an angle that is $\frac{4}{5}$ of a straight angle.
4. Draw an angle that is $\frac{3}{2}$ of a straight angle.
5. Draw an angle that is $\frac{5}{4}$ of a straight angle.
6. Angle $\hat{\alpha}$ is three times angle $\hat{\beta}$. Angles $\hat{\alpha}$ and $\hat{\beta}$ are also supplementary. Can you find what they measure?

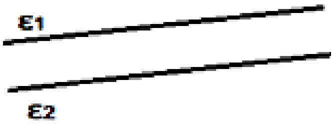
Activity A.6.3

A) Draw a circle on your notebook and divide it into **6 equal arcs**. Compare the radius of the circle with the string of each arc. What have you noticed?

B) Based on what you just noticed in question (A), think: can we divide a circle into 6 equal parts, **without** using a protractor?

A.7 – Line & Line

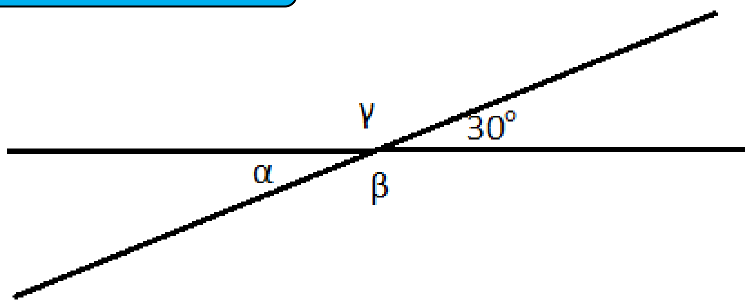




Two lines that are on the same plane can either **intersect** (they are **intersecting lines**), or they do **do not intersect** no matter how long we extend them. These lines are called **parallel lines.** We use the notation

$\varepsilon_1 // \varepsilon_2$ to show that two lines are parallel.

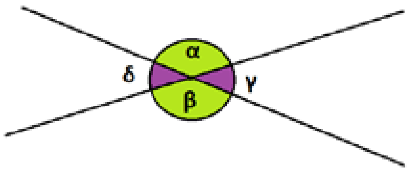
Άσκηση A7.1



1. $\hat{\gamma} + 30^0 =$ ……………, so $\hat{\gamma} =$ …………….
2. $\hat{\beta} + 30^0 =$ ……………., so $\hat{\beta} =$ …………..
3. Can you measure $\hat{\alpha}$?
4. What have you noticed?

Two intersecting lines always define four angles.

- The adjacent angles (the ones that are next to each other), are ……………………………….. because together they form a straight angle.
- The ones opposite to each other (i.e., $\hat{\alpha}$ & $\hat{\beta}$ and $\hat{\gamma}$ & $\hat{\delta}$ are **vertically opposite angles.**



Complete

Angles $\widehat{\alpha}$ & $\widehat{\beta}$ are vertically opposite; they are …………

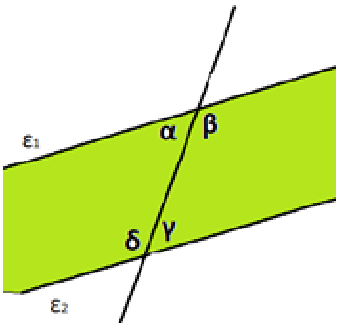
Angles $\widehat{\gamma}$ & $\widehat{\delta}$ are ………………………..; they are …………

Activity A7.2

Complete the table:

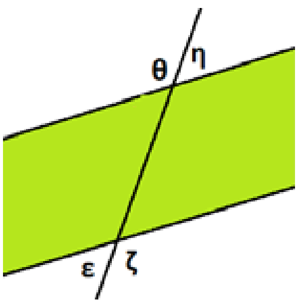
Angle $\widehat{\omega}$	Complementary $\widehat{\omega}$	Supplementary to $\widehat{\omega}$
45°		
120°		
90°		
12°		
180°		

Angles created when two parallel lines are intersected by a third line



Lines ε_1 and ε_1 are parallel ($\varepsilon_1 // \varepsilon_2$) and line ε intersects them. This creates 8 angles.

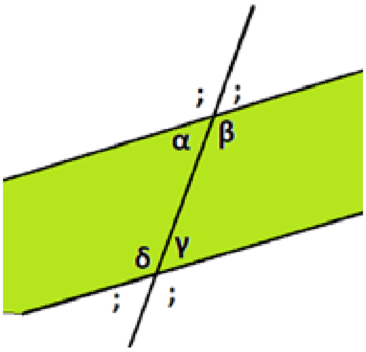
Four of these angles are *inside the zone* defined by the parallels (green area). We call these **interior**[3] angles. These are ***angles*** $\widehat{\alpha}, \widehat{\beta}, \widehat{\gamma}$ and $\widehat{\delta}$.



The other four angles are *outside the zone* (white area). We call these **exterior**[4] angles. These are **angles** $\widehat{\varepsilon}, \widehat{\zeta}, \widehat{\eta}$ and $\widehat{\theta}$.

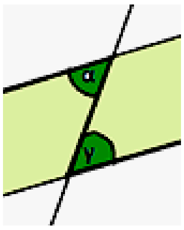
Each one of these is *vertically opposite* to an interior angle, so it is equal to it.

[3] Interior = inside.
[4] Exterior = outside.



Activity A.7.1

We can show that two angles in a figure are equal by giving them the same name. Can you complete the figure by naming the ***exterior*** angles? (This name should be the same as the name of an equal angle)

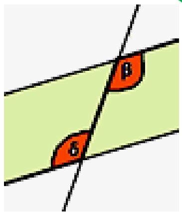


Angles $\hat{\alpha}$ and $\hat{\gamma}$ are **interior** alternate and are **equal**.

$$\hat{\alpha} = \hat{\gamma}$$

Angles $\hat{\beta}$ and $\hat{\delta}$ are **exterior** alternate and are **equal.**

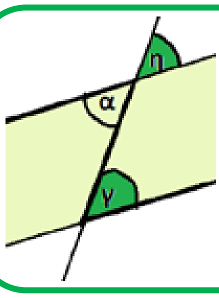
$$\hat{\beta} = \hat{\delta}$$



Angles $\hat{\alpha}$ and $\hat{\eta}$ are vertically oppite and are equal. $\hat{\alpha} = \hat{\eta}$

Angles $\hat{\alpha}$ and $\hat{\gamma}$ are **interior** alternate and are equal. $\hat{\alpha} = \hat{\gamma}$

Therefore $\hat{\gamma} = \hat{\eta}$



Angles $\hat{\gamma}$ and $\hat{\eta}$ are corresponding angles and they are equal.

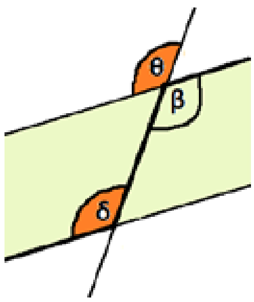
$$\hat{\gamma} = \hat{\eta}$$

***Exercise A.7.1*:** Complete the following sentences.

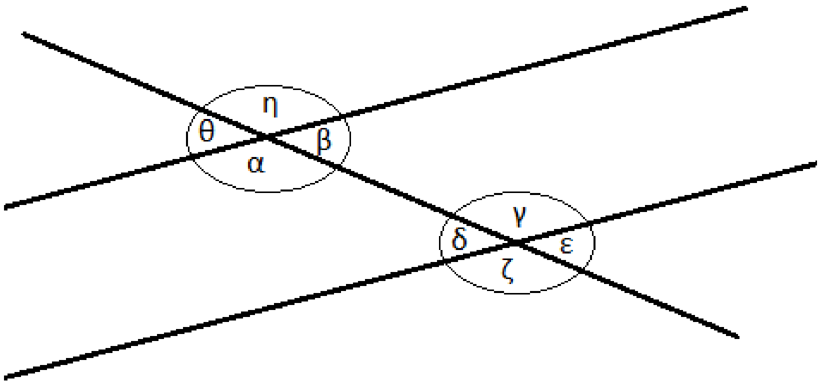
Angles $\hat{\delta}$ and $\hat{\theta}$ are called ……………………………………………………………
and they are ……………………………

Angles $\hat{\delta}$ and $\hat{\beta}$ are called ……………………………………………………………
and they are ……………………………

Angles $\hat{\theta}$ and $\hat{\beta}$ are called ……………………………………………………………
and they are ……………………………



Activity A.7.2*:* Let's colour all *equal angles* ***using the same colour.*** How many colours will we need?



There are 4 acute + 4 obtuse angles = 8 angles.

All the acute angles in the figure are ………………. and all the obtuse angles are also …………….

An acute angle and an obtuse angle in this figure are ………………….………….…

Drawing a line parallel to another line

Draw a line ε. Next draw another line that is **parallel** to ε. Πώς How can you be certain that the line you drew was truly parallel? The sentence below will help you find the way.

If two lines are perpendicular to the same line, that makes them parallel.

My new words

……

……

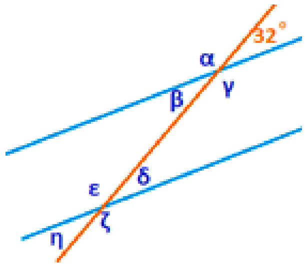
……

……

……

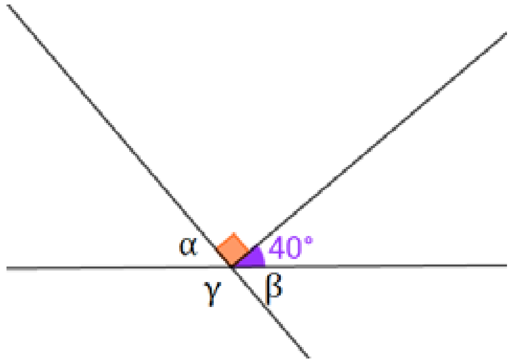
Exercise A.7.1

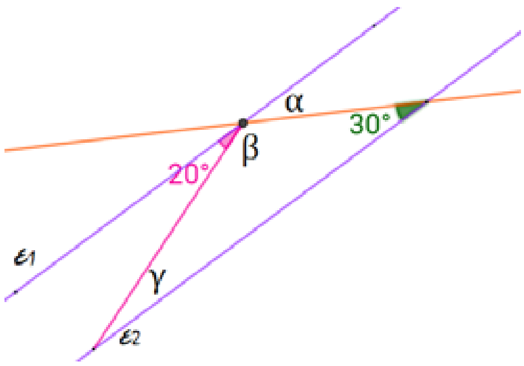
Estimate angles $\hat{\alpha}, \hat{\beta}, \hat{\gamma}, \hat{\delta}, \hat{\varepsilon}, \hat{\zeta}$ and $\hat{\eta}$ in this figure.



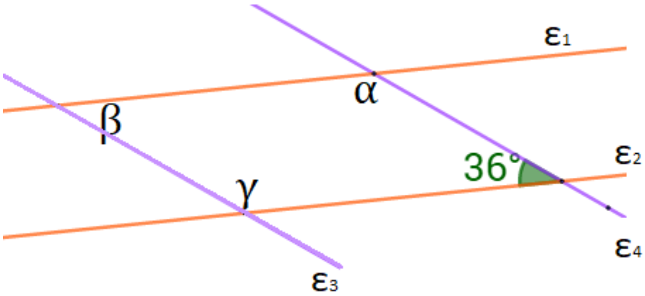
Exercises

Can you estimate angles $\hat{\alpha}, \hat{\beta}$ and $\hat{\gamma}$ in the figures below?





Lines ε_1 and ε_2 are parallel ($\varepsilon_1 // \varepsilon_2$)



$\varepsilon_1 // \varepsilon_2$

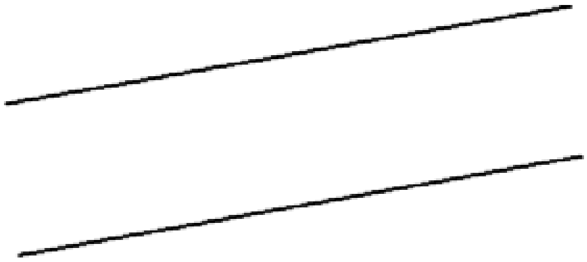
and

$\varepsilon_3 // \varepsilon_4$

Distance between two parallel lines

Activity A7.3

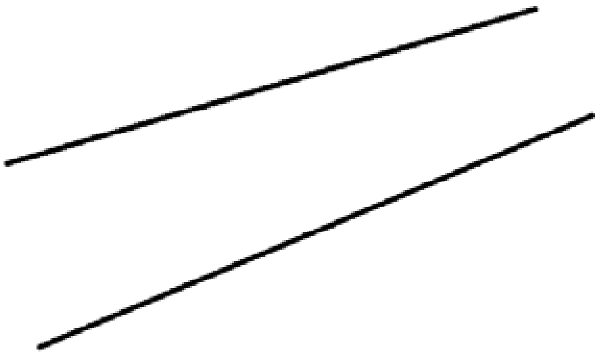
Halil and Mariam measured how wide the road in front of their home is, but they got different results. This problem is just like measuring the distance between two parallel lines.



How would you find the distance between two parallel lines? Discuss this with your group. Will this distance change if we extend the line segments that are shown?

Activity A7.4

Are the lines in this figure parallel or not? How did you figure this out?

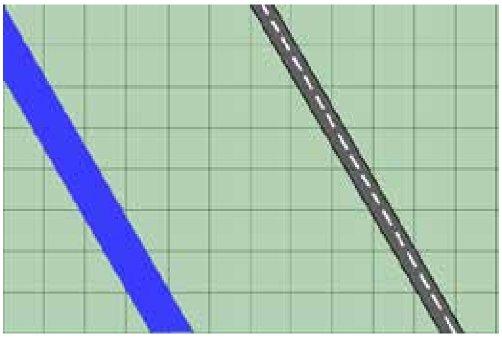


If two lines are parallel, the distance between them is ………………………………..

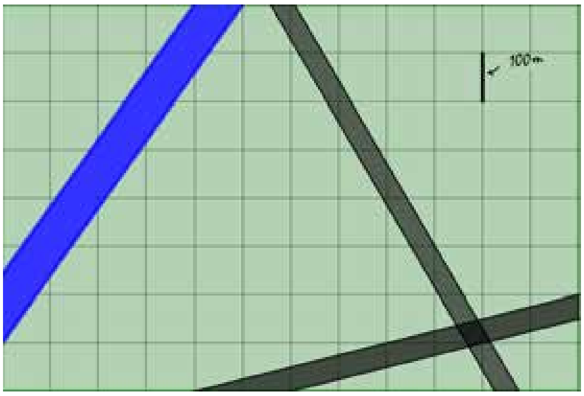
To find the distance between two parallel lines we measure
……

Activity A7.5

Fanis wants to walk **parallel** to the river and the road. He also wants to keep the same distance from both. Can you draw the line he needs to follow?



Activity A7.6



Halil and Mariam want to set up a tent that is more than 200m from the river and more than 300m from each road. Can you show them where to set up their tent?

A.8 – Lines & Circles

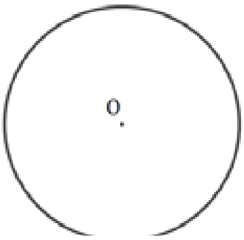


Figure 1

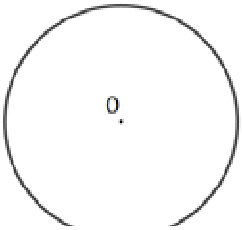


Figure 2

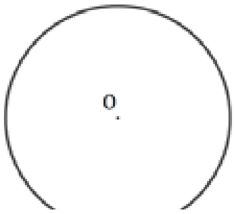


Figure 3

A) How many points can a line have in common with a circle? Could there be three share points?

B) Draw a line ε which has **two points in common** with the circle (Figure 1).

C) Draw a line ε which has **just one point in common** with the circle (Figure 2).

D) Draw a line ε which has **no points in common** with the circle (Figure 3).

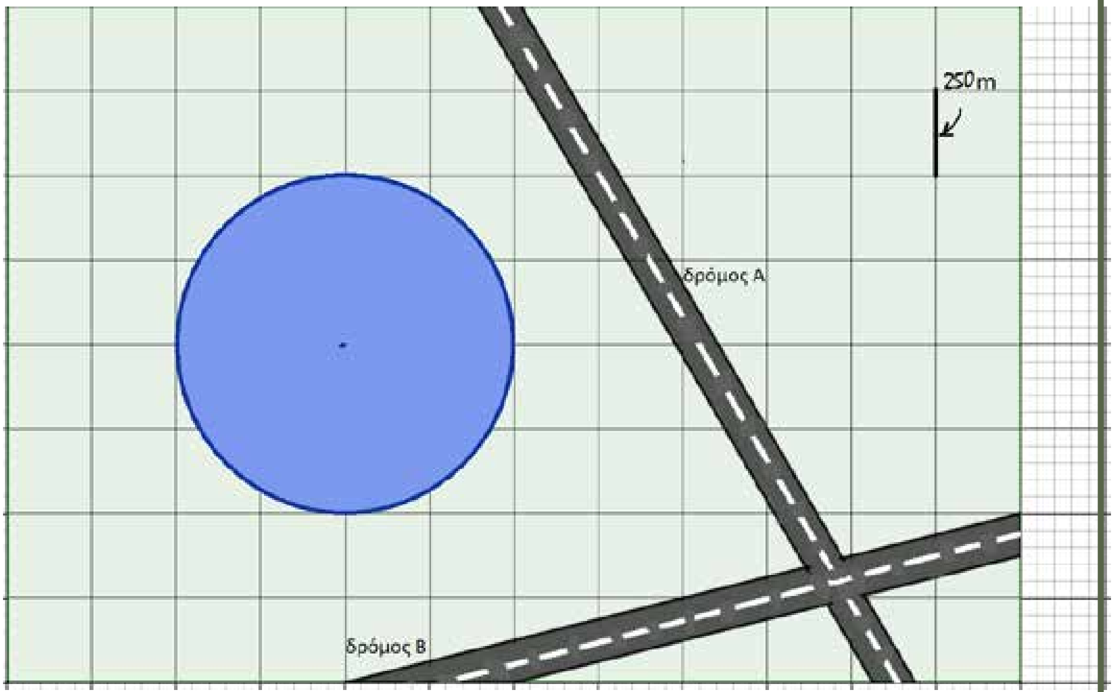
E) In each case draw the distance from the centre O to the line ε d(O,ε).

The line in figure 1 is a **secant line**; d(O,ε) …….. ρ (choose <, > ή =)

The line in figure 2 is a **tangent line**; d(O,ε) …….. ρ (choose <, > ή =)

The line in figure 3 is **external line**; d(O,ε) …….. ρ (choose <, > ή =)

Activity A8.1



A. Which parts of the lake (blue circle) have a distance greater than 1km from the road B?

B. Which part of the lake is closer a) to road A (δρόμος A) b) to road B (δρόμος B)?

C. Where on the lake should someone be, if they want to be at a distance *greater than 750m* from road A?

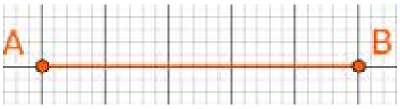
D. Nadia wants to build a shop that is:

- More than 400m but less than 800m from the lake,
- More than 400m from road A
- More than 350m from road B.

Could you colour the suitable area for her?

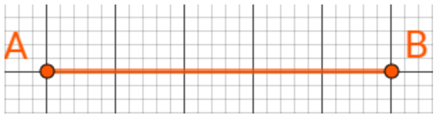
A.9 – Circle & circle

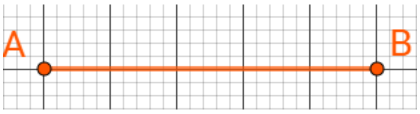
A line segment AB has a length of 5.



A) Draw the circle (A, 4) and the circle (B, 3).

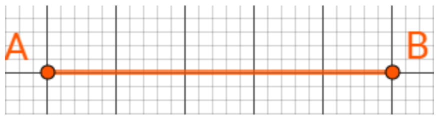
B) Draw the circle (A,3) and the circle (B,2).

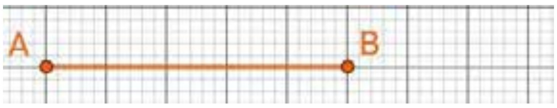




C) Draw the circles (A, 5) and (B, 2).

D) Draw the circles (A,2) and (B,1).





E) Draw the circles (A,8) and (B,2).

How many points does each circle have in common in the other? Can they have three points in common?

Possibility A: two circles **intersect**. Then, they have ………. (how many?) points in common.

Possibility B: two circles **touch each other ……………..** and have ……… point in common.

Possibility C: two circles **touch each other ………………** and have ………. point in common.

Possibility D: one circle is **external to the other** and ………………….

Possibility E: one circle is **internal to the other** and ………………….

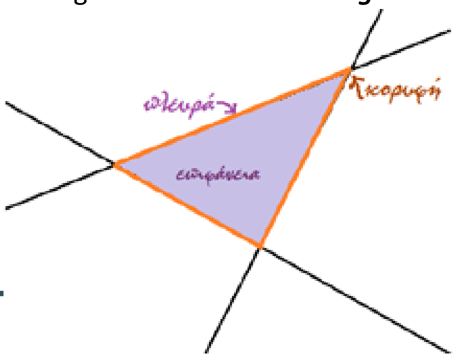
For each possibility, ***compare*** the length of AB (the line segment that connects the centres of the circles) with the sum $\rho_1+\rho_2$ or the difference $\rho_1-\rho_2$. Find out **when: a) do the two circles intersect? b) do the two circles touch each other internally and c) is one circle external to the other?**

A.10 – Triangles

Activity A.10.1 (Discuss with your group)

Draw 3 lines. In how many ways can you do this? Which of these do you see more usually around you?

When 3 lines intersect each other in sets of two, they define a figure that we call a ***triangle.*** This figure has ***3 sides (πλευρές)***, ***3 angles (γωνίες), 3 apexes (κορυφές)*** and ***a surface (επιφάνεια)*** (the part of a plane that is enclosed by the triangle's sides).



A triangle is the simplest flat ***shape***. It is one of many ***polygons***. It has the smallest number of sides.

Activity A.10.2 (Discuss with your group)

Fanis once found some sticks on the ground. He started playing with them and build shapes. First, he built a triangle. But he needed more sticks, so he broke some. But then, when he tied to make another triangle, he couldn't! So, he started thinking why…

He soon reached a conclusion. **Can you figure out what he discovered**?

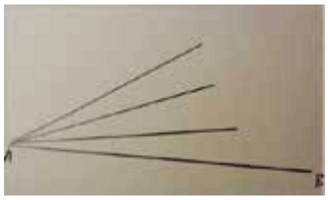
If you don't have sticks, you can use anything else, like straws, pencils, etc.

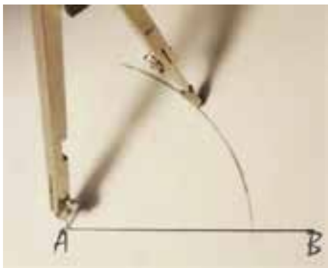
But we can also think in geometrical terms, and use some of the tools we have in geometry, like a ruler and a compass. The problem we are dealing with is the same as the following.

Activity A.10.3: Build a triangle with sides α, β and c.

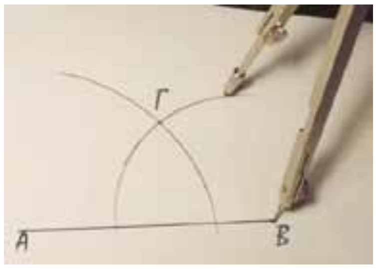
Suppose we want to draw a triangle with sides as follows **A = 5cm, b = 6 cm** and **c = 8cm**? How can we go about doing this?

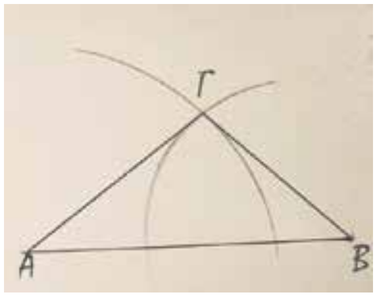
It is easy to draw one of the sides. Let's start with the longest one, and call it AB. After that, we want to draw the following one, AΓ. But we don't know where … So we draw all the ***possible positions that have a distance from A equal to the one we need.***





In other words, we're drawing a circle! Its centre is A and its radius is equal to b = 6cm. Next we need a circle with a centre on B and a radius equal to a = 5cm. Their intersection is the third apex of the triangle. This point is 6cm from apex A and 5cm from apex B. The triangle we wanted to draw is AΒΓ.





Now draw a triangle with the following sides:

A) a = 8cm, b = 4cm and c = 3cm.

B) a = 8cm, b = 5cm and c = 3cm.

What have you noticed? Have you drawn a conclusion?

Exercise A.10.1

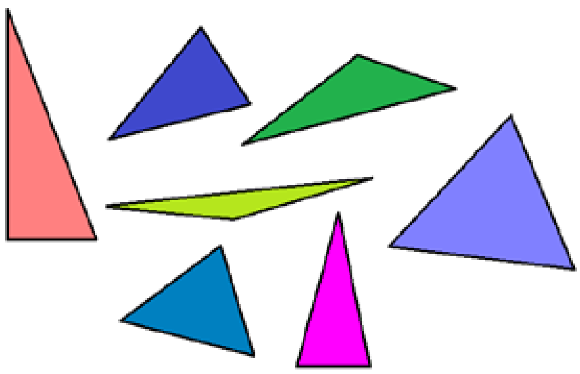
Check if there is a triangle with the following side measurements:

a) a = 5cm, b = 3cm and c = 3cm   b) a = 5cm, b = 5cm and c = 5cm

b) a = 7cm, b = 4cm and c = 3cm d) a = 6cm, b = 3cm and c = 2cm

e) a = 7cm, b = 4cm and c = 4cm

If here is one, <u>draw it</u>.



You might notice that there are different types of triangles.

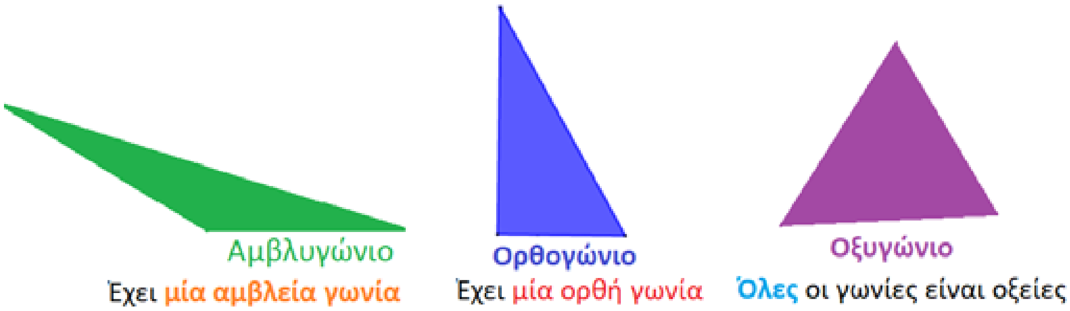
Mathematicians have classified triangles according to

a) their sides   b) their angles.

Classification of triangles according to sides

Equilateral	Isosceles	Scalene
All the sides are equal	Two sides are equal	All the sides are unequal

Classification of triangles according to angles



Obtuse angled triangle
It has one obtuse angle

Right triangle
It has one right angle

Acute angled triangle
All the angles are acute

Exercise A.10.2: Draw an isosceles triangle

A) Draw a line segment AB measuring 3cm.

B) Draw the circle (A, 5cm) and (B, 5cm).

C) Name the intersections of the circles, Γ and Δ.

D) Draw the line segments ΑΓ, ΒΓ and ΑΔ, ΒΔ.

What kind of triangle is ΑΒΓ? What kind of triangle is ΑΒΔ?

Exercise A.10.3: Draw an equilateral triangle

A) Draw an equilateral triangle with sides measuring a = 5cm.

B) Draw an equilateral triangle of with a side of random length.

Respond to the questions

1. Is there a triangle with **two** obtuse angles?
   If so, draw it. If not, explain why.

2. Is there a triangle with **two** right angles?
   If so, draw it. If not, explain why.

Activity A.10.4 Types of triangles

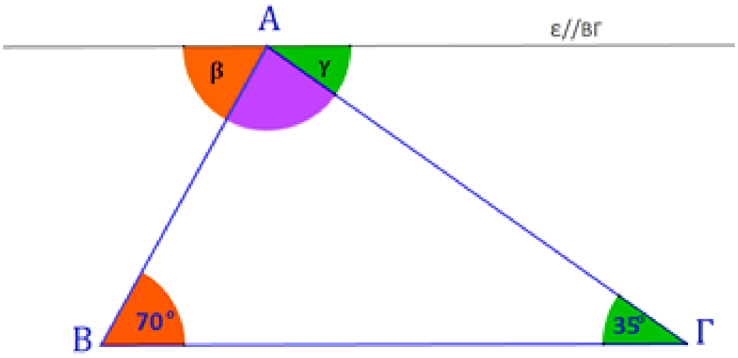
— *Can't a triangle belong to two categories at the same time?* Nadia wondered.
— *What do you mean?* Halil asked.
— *You know, like can triangle be **both** isosceles **and** right.*
— *Hm...* Halil thought for a moment. *I think it's possible. But I'm not sure all combinations are possible.*

The children considered all the combinations using a table. They also tried to draw them. They eventually decided that some of these **cannot exist.** They ticked (√) the boxes that corresponded to triangles that exist, and they crossed (X) the boxes that corresponded to triangles that don't exist. How about you? Can you figure out which exist and which don't? Draw one triangle of each kind that exists.

	Scalene	Isosceles	Equilateral
Acute angled			
Right angled			
Obtuse angled			

A.11 – Adding the angles of a triangle

Activity A.11.1



In this figure, line ε είναι is parallel to the triangle's base (ΒΓ).

We know the two angles of the triangle, $\hat{B} = 70^o$ and $\hat{\Gamma} = 35^o$.

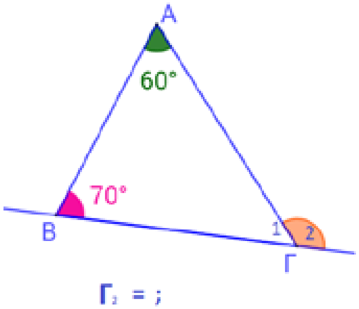
1. What is the relationship between angle $\hat{B}$ of the triangle and $\hat{\beta}$? Can you estimate angle $\hat{\beta}$?
2. What is the relationship between angle $\hat{\Gamma}$ of the triangle and $\hat{\gamma}$? Can you estimate angle $\hat{\gamma}$?
3. $\hat{A}$+ $\hat{\beta}$ + $\hat{\gamma}$ = ………………… Can you estimate angle $\hat{A}$? of the triangle.
4. What do you conclude about the sum of a triangle's angles $\hat{A} + \hat{B} + \hat{\Gamma}$?

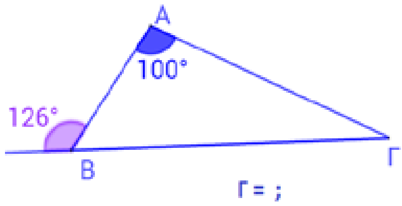
Think: Is the sum a triangle's angles $\hat{A} + \hat{B} + \hat{\Gamma}$ different if the angles B and Γ are different?

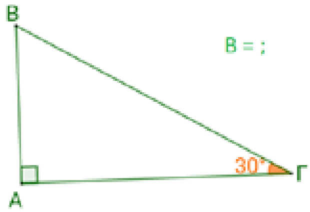
In a triangle ΑΒΓ, the sum of the angles is $\hat{A} + \hat{B} + \hat{\Gamma}$ = ……… …..……………

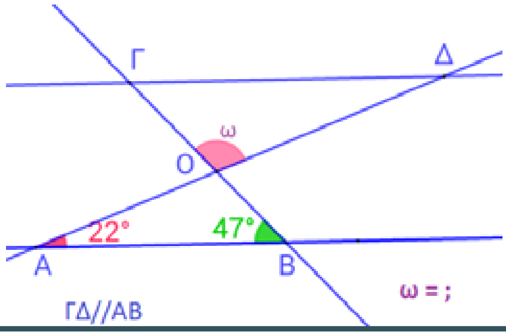
Exercise A.11.1

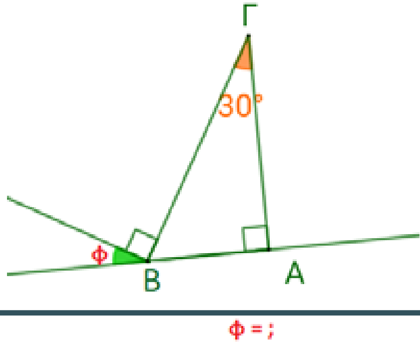
In the following shapes, find the angles that are requested









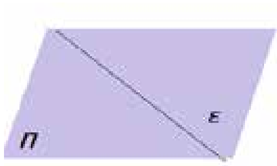


Part B

Planes

A plane: we can imagine a plane by thinking about the wall of a house, the page of a notebook, or the surface of a table.

Every flat[5] shape has **length** and **width,** but it has **no height.** We use capital letters to name planes. For example. we can talk about ***plane Σ.***



Lines divide planes into two half planes.

Activity (groupwork)

- Draw two points (A and B) on your notebook; draw line AB.
- Draw a plane that passes through these two points.
- Can you draw another plane that crosses A and B?
- How many planes do you think might cross A and B?
- Draw a point C which is **not** on line AB. How many planes do you think might cross these **three non-collinear**[6] **points (A, B,** and **Γ)?**

Complete & Learn

There .... crossing two points A and B

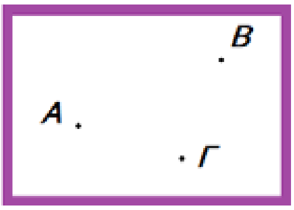
There ....crossing three points that are not collinear

[5] Flat shapes: shapes on a plane
[6] Collinear points: points that are on the same line.

Activity

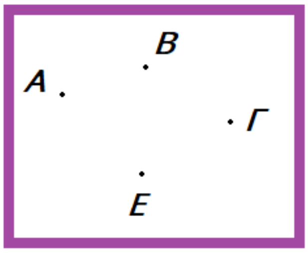
Discuss the following questions with your group, decide together, draw and write…

- ALL the line segments that are defined by ***2 of the 3*** points A, B and Γ.



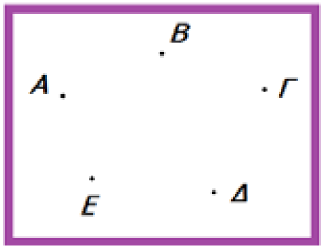
Number= **Names:**

- ALL the line segments that are defined by ***2 of the 4*** non-collinear points A, B, Γ and Δ.



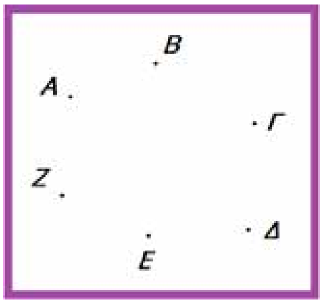
Number= **Names:**

- ALL the line segments that are defined by ***2 of the 5*** non-collinear points A, B, Γ, Δ and E.



Number= **Names:**

- ALL the line segments that are defined by ***2 of the 6*** non-collinear points A, B, Γ, Δ, E and Z.

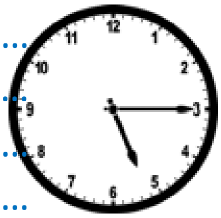


Number= **Names:**

Can you **predict** how many line segments might exist that are defined by 2 out of 7 non-collinear points? Explain your thought process.

Activity: identify the type of an angle

Write down the **type** of the convex angle that the hands of the clock shape, when it's:



5:15 ....

2:20 ....

4:50 ....

6:00 ....

12:00 ....

5:00 ....

4:55 ....

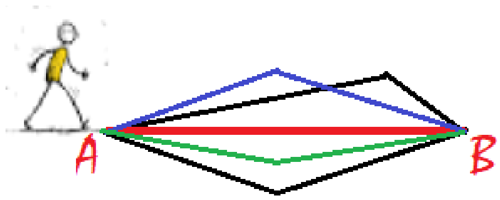
Unequal triangles

What happens with any **triangle with sides measuring 3 numbers a, b and c** is that:

Every side of the triangle is larger than the sum of the other two.

$$a < b + c \; , \quad b < a + c \quad \text{και} \quad c < a + c$$

This is something we already knew: the *shortest distance* from one point (A) to another point (B) is the line segment AB.

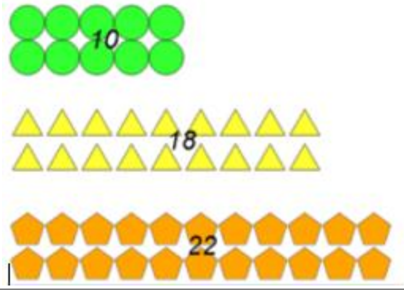
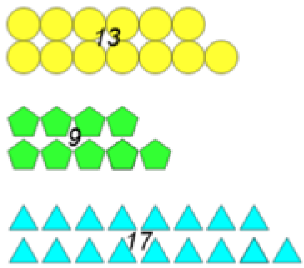
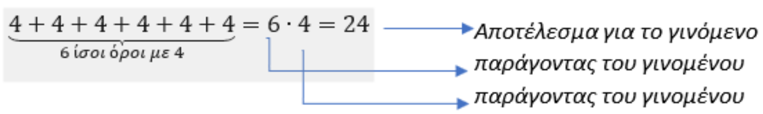
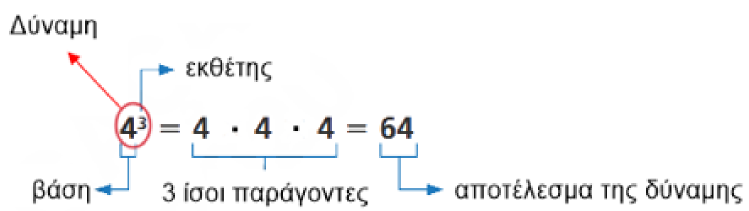


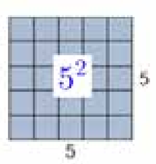
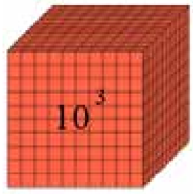
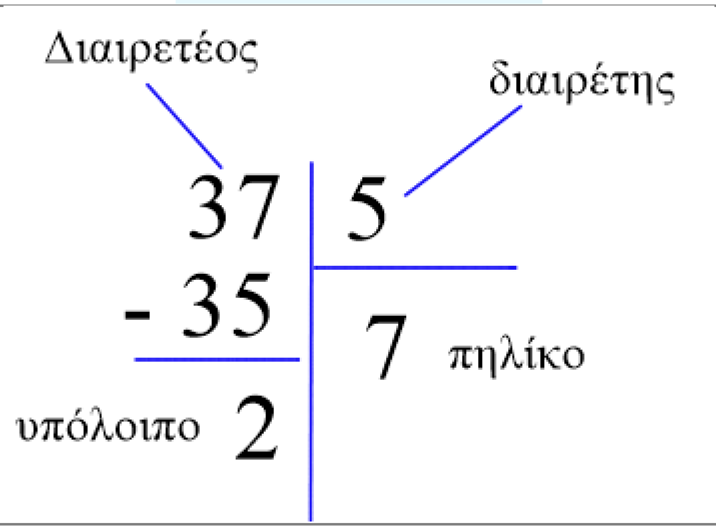
So how do we know **if a triangle exists** with sides a, b and c? Yannis figured out that we can be certain, ***if we check that the largest side is smaller than the sum of the other two***. Is he right? Discuss this with your group.

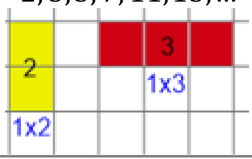
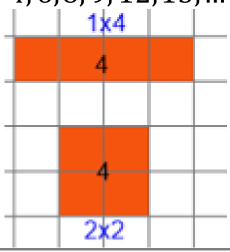
Useful links about:

1. Comparing line segments (Geogebra):
   http://photodentro.edu.gr/aggregator/lo/photodentro-lor-8521-5714
2. Adding line segments (Geogebra):
   http://photodentro.edu.gr/aggregator/lo/photodentro-lor-8521-5715
3. Lines, half lines, and line segments (Geogebra):
   http://photodentro.edu.gr/aggregator/lo/photodentro-lor-8521-2365
4. The concept of an angle (Geogebra): http://photodentro.edu.gr/v/item/ds/14354
5. Consecutive and adjacent angles (Geogebra):
   http://photodentro.edu.gr/lor/r/8521/2184
6. Adjacent and explementary angles: (Χελωνόκοσμος)
   http://photodentro.edu.gr/aggregator/lo/photodentro-lor-8521-9520
7. Vertically opposite angles (Χελωνόκοσμος):
   http://photodentro.edu.gr/aggregator/lo/photodentro-lor-8521-9521
8. Distance between a point and a line (Geogebra):
   http://photodentro.edu.gr/aggregator/lo/photodentro-lor-8521-2145
9. Relative positions of line and circle (Geogebra):
   http://photodentro.edu.gr/aggregator/lo/photodentro-lor-8521-2166
10. Investigating overlapping circles (Geogebra):
    http://photodentro.edu.gr/aggregator/lo/photodentro-lor-8521-2086
11. Unequal triangles (Geogebra): http://photodentro.edu.gr/aggregator/lo/photodentro-lor-8521-5860

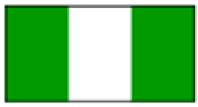
Glossary with examples

Natural numbers		
Concept	**Example**	**Page**
Natural numbers	$0, 1, 2, 3, \dots, 29, 30, \dots, 999, 1000, \dots$	
Even numbers	$0, 2, 4, 6, 8, 10, 12, \dots$ 	
Odd numbers	$1, 3, 5, 7, 9, 11, \dots$ 	
Addition - sum	10+5=15	
Αφαίρεση – difference	$12 - 2 = 10$	
Multiplication – product – factors		*Six terms equal to four* *Product (result)* *Factor of the product* *Factor of the product*
Power	$3^4 = 3 \cdot 3 \cdot 3 \cdot 3$	
Base - exponent		*Power* *Base* *Exponent*

		Three equal factors *Result of the power*
Square of a number	$2^2, 7^2, 123^2$ 	
Cube of a number	$2^3, 6^3, 304^3$ 	
Division - quotient	$15:3 = 5$ 	
divisor – dividend – remainder		*Dividend* *divisor* *remainder* *quotient*
Euclidean division (division with remainder)	$37 = 5 \cdot 7 + 2$	
Natural number multiples	The multiples of 5 are: $0, 5, 10, 15, 20, \ldots$	

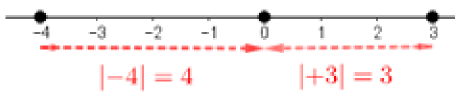
Least (: smallest) Common Multiple, LCM	The multiples of 5 are: 0,5,10,15,10,… The multiples of 3 are: 0, 3, 6, 9, 12, 15, 18, … $LCM(5,3) = 15$	
Natural number divisors	The divisors of 15 are: $1, 3, 5, 15$	
Greatest (: largest) common divisor	The divisors of 15 are: $1, 3, 5, 15$ The divisors of 12 are: 1, 2, 3, 4, 6, 12 $GCD(15,12) = 3$	
Prime numbers	$2, 3, 5, 7, 11, 13, \ldots$  	
Composite numbers	$4, 6, 8, 9, 12, 15, \ldots$  	

Glossary with Examples

Fractions		
Concept	**Example**	**Page**
Fraction	Fraction: $\frac{2}{3}$ ← numerator, fraction bar, demoninator  We read this "two thirds" or "two over three"	1
Equivalent fractions	$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{6}{12}$	4
Reducing/simplifying fractions	$\frac{18}{27} = \frac{18:9}{27:9} = \frac{2}{3}$	6
Irreducible fractions	These are irreducible: $\frac{1}{2}, \frac{3}{4}, \frac{7}{5}$ These are not irreducible: $\frac{2}{4}, \frac{9}{12}, \frac{14}{10}$	6
Similar fractions	$\frac{3}{5}, \frac{12}{5}$ $\frac{5}{7}, \frac{3}{7}$ $\frac{6}{13}, \frac{17}{13}$	7
Dissimilar fractions	$\frac{3}{5}, \frac{12}{7}$ $\frac{5}{8}, \frac{3}{7}$ $\frac{6}{13}, \frac{17}{14}$	8
Adding fractions	$\frac{\alpha}{\beta} + \frac{\gamma}{\beta} = \frac{\alpha+\gamma}{\beta}$ $\frac{5}{3} + \frac{2}{3} = \frac{5+2}{3} = \frac{7}{3},$ $\frac{2}{3} + \frac{1}{2} = \frac{4}{6} + \frac{3}{6} = \frac{4+3}{6} = \frac{7}{6}$	12
Subtracting fractions	$\frac{\alpha}{\beta} - \frac{\gamma}{\beta} = \frac{\alpha-\gamma}{\beta}$ $\frac{5}{3} - \frac{2}{3} = \frac{5-2}{3} = \frac{3}{3} = 1,$ $\frac{2}{3} - \frac{1}{2} = \frac{4}{6} - \frac{3}{6} = \frac{4-3}{6} = \frac{1}{6}$	12

Multiplying fractions	$\alpha \cdot \frac{\beta}{\gamma} = \frac{\alpha \cdot \beta}{\gamma}$, π.χ. $5 \cdot \frac{2}{3} = \frac{5 \cdot 2}{3} = \frac{10}{3}$ $\frac{\alpha}{\beta} \cdot \frac{\gamma}{\delta} = \frac{\alpha \cdot \gamma}{\beta \cdot \delta}$, π.χ. $\frac{4}{5} \cdot \frac{2}{3} = \frac{4 \cdot 2}{5 \cdot 3} = \frac{8}{15}$	17
Reciprocal numbers	$\frac{3}{4}$ with $\frac{4}{3}$, 2 with $\frac{1}{2}$, α with $\frac{1}{\alpha}$ $2 = \frac{2}{1} \rightarrow \frac{1}{2}$ $\frac{3}{4} \rightarrow \frac{4}{3}$ because: $\frac{3}{4} \cdot \frac{4}{3} = 1$, $2 \cdot \frac{1}{2} = 1$, $\alpha \cdot \frac{1}{\alpha} = 1$	18
Dividing fractions	$\frac{\alpha}{\beta} : \frac{\gamma}{\delta} = \frac{\alpha}{\beta} \cdot \frac{\delta}{\gamma} = \frac{\alpha \cdot \delta}{\beta \cdot \gamma}$, π.χ. $\frac{4}{5} : \frac{2}{3} = \frac{4}{5} \cdot \frac{3}{2} \; \frac{4 \cdot 3}{5 \cdot 2} = \frac{6}{5}$	19

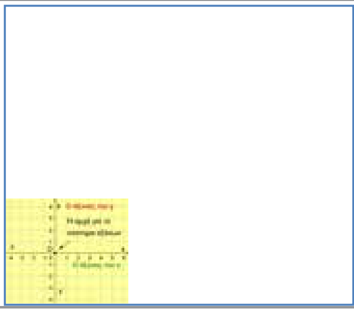
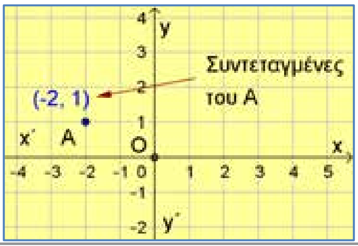
Glossary with examples

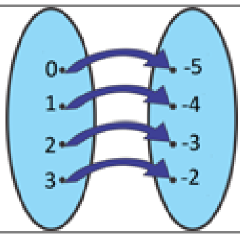
Integers and operations		
Concept	**Example**	**Page**
natural numbers or positive numbers	0, 1, 2, 3, 4, 5, ..., 1000, ...	2
negative numbers	-1, -2, -3, -4, ..., -1000, ...	2
sign	+4, -7, +9, -122	2
number line		2
same sign (like) numbers	2 and 5, -6 and -4,	2
opposite sign (unlike) numbers	2 and -4, -9 and 7	2
opposite numbers	5 and -5, -7 and 7, -123 and 123	3
integers	..., -1000, ..., -4, -3, -2, -1, 0, 1, 2, 3, 4, ..., 1000,...	3
absolute value of number		4

Glossary with examples

Algebra: Εξίσωση 1ου βαθμού		
Concept	**Example**	**Page**
Equality	$7 - 3 = 8 \div 2$	2
Linear (degree one) equation with one unknown	$5x - 6 = 10 - 3x$	3
Solution/root of an equation	2 solves (is the solution of) $5x - 6 = 10 - 3x$	4
Algebra: Ανίσωση 1ου βαθμού		
Concept	**Example**	**Page**
inequality	$7 - 3 > 2$, $3+1 < 2 \cdot 5$	1
Linear (degree one) inequality with one unknown	$8x - 3 > 12 + 2x$	6
Solution/root of an inequality	3, 4, 5, 12 solve (are solutions for) $8x - 3 > 12 + 2x$	7

Glossary with examples

<table>
<tr><th colspan="3">Functions</th></tr>
<tr><th>Concept</th><th>Example</th><th>Page</th></tr>
<tr><td>Cartesian coordinate plane
• x-axis
• y-axis
• coordinates plane point zero</td><td></td><td>1</td></tr>
<tr><td>coordinates</td><td>
</td><td>2</td></tr>
<tr><td>x-value</td><td>(-4, 5) , (3, 0) , (0, -2) , (1, -100)</td><td>2</td></tr>
<tr><td>y-value</td><td>(-4, 5) , (3, 0) , (0, -2) , (1, -100)</td><td>2</td></tr>
<tr><td>proportional values</td><td>$\frac{y}{x} = 5$
<table>
<tr><th>x</th><th>y</th></tr>
<tr><td>1</td><td>5</td></tr>
<tr><td>2</td><td>10</td></tr>
<tr><td>6</td><td>30</td></tr>
<tr><td>18</td><td>90</td></tr>
</table>
x 2   x 2
x 3   x 3</td><td>4</td></tr>
<tr><td>proportionality coefficient</td><td>$\frac{y}{x} = a$</td><td>4</td></tr>
<tr><td>inversely proportional values</td><td>x 2
<table>
<tr><th>x</th><th>y</th></tr>
<tr><td>1</td><td>12</td></tr>
<tr><td>2</td><td>6</td></tr>
<tr><td>3</td><td>4</td></tr>
</table>
: 2
$x \cdot y = 12$</td><td>8</td></tr>
<tr><td>values tale</td><td>
<table>
<tr><th>x</th><td>-1</td><td>0</td><td>1</td><td>2</td><td>3</td><td>6</td></tr>
<tr><th>y</th><td>-2</td><td>0</td><td>2</td><td>4</td><td>6</td><td>12</td></tr>
</table></td><td>12</td></tr>
</table>

<table>
<tr><td>diagram</td><td>
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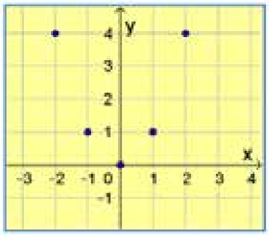
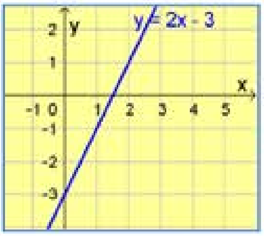
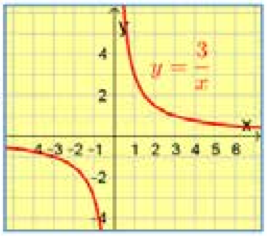
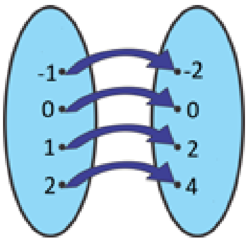
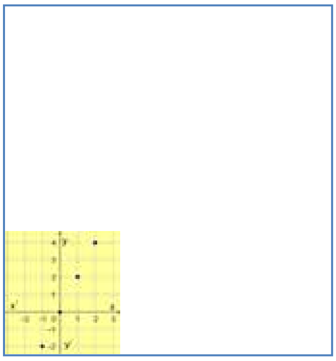
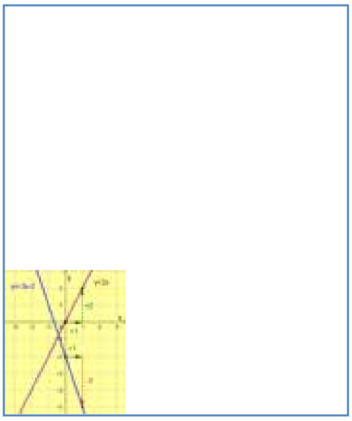
<tr><td>graph</td><td>


</td><td>12</td></tr>
<tr><td>formula</td><td>y=-2x, y=2x-1, y=x², y=$\frac{3}{x}$</td><td>12</td></tr>
<tr><td>function</td><td>
<table>
<tr><td>x</td><td>-1</td><td>0</td><td>1</td><td>2</td></tr>
<tr><td>y</td><td>-2</td><td>0</td><td>2</td><td>4</td></tr>
</table>


y= 2x
«times two»
</td><td>11</td></tr>
<tr><td>line gradient</td><td>y= 2x , y=-3x+4
</td><td>15, 19</td></tr>
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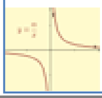
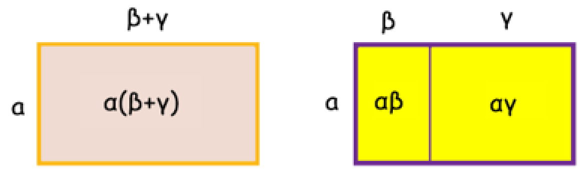
hyperbola		22

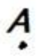
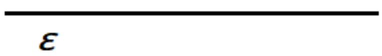
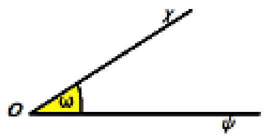
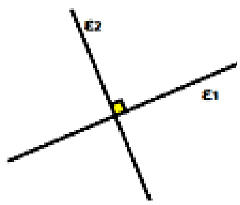
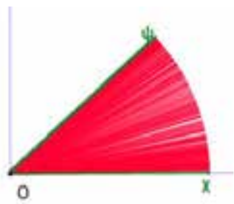
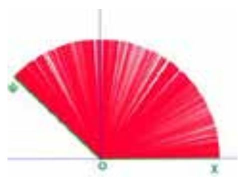
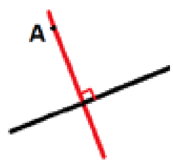
Glossary with examples

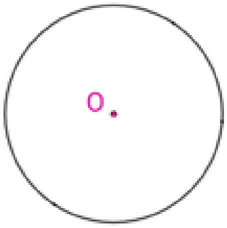
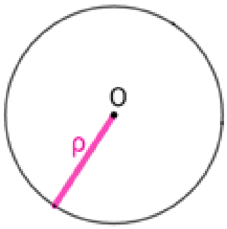
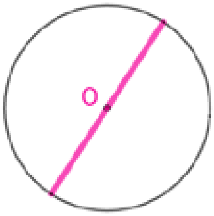
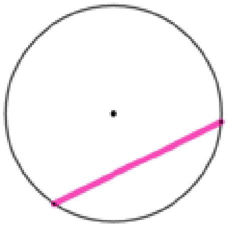
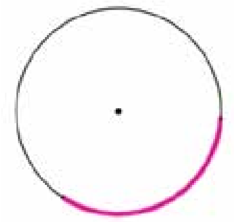
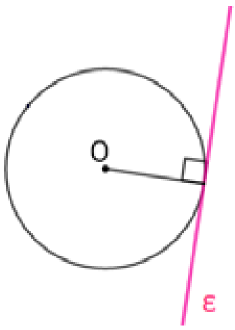
Algebra: algebraic formulas		
Concept	**Example**	**Page**
Variable	κ x a	2
Algebraic formula	$3x$ $x+5$ $y^2-2y+\frac{1}{6}$ $3\omega^2\alpha\beta^3$	3
Numerical values for algebraic formula	For $x=2$, $is: x+5=2+5=7$	3
similar terms [1]	$3\alpha+2\beta-\alpha-5\beta+6\alpha$ The following sets of terms are like: $3\alpha, -\alpha, 6\alpha$ and: $2\beta, -5\beta$	6
Distributive property	$\boldsymbol{\alpha(\beta+\gamma)=\alpha\beta+\alpha\gamma}$  $\beta+\gamma$ $\begin{array}{c\|c\|c} \cdot & \beta & \gamma \\ \hline \alpha & \alpha\beta & \alpha\gamma \end{array}$	9
Development of a product	The product $\alpha(\beta+\gamma)$ is developed as $\alpha\beta+\alpha\gamma$	9
Monomial	$3x^2$ $-31xy$ $\frac{1}{3}x^4y^3$ $-x$ 5	12
Monomial main part & coefficient	$-2\,x^2y$ *Κύριο μέρος* *Συντελεστής* Main part Coefficient	12
Similar monomials	They have the same main part $2\alpha^2\beta, -\alpha^2\beta, \frac{1}{2}\alpha^2\beta$	12
Polynomial	$3x^2-5x$	12

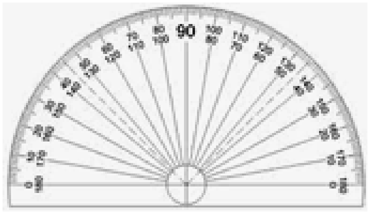
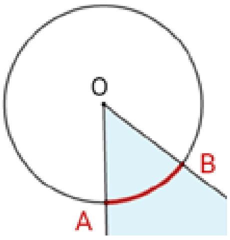
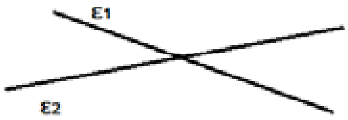
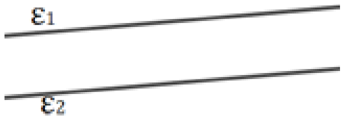
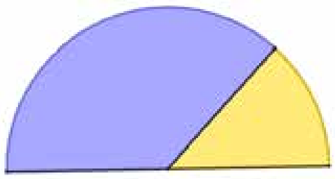
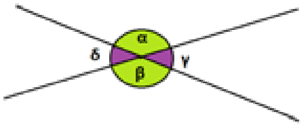
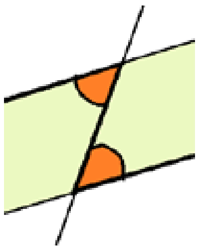
[1] *See the similar monomials below*

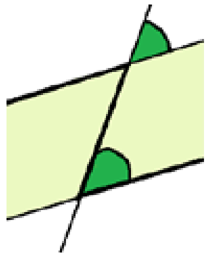
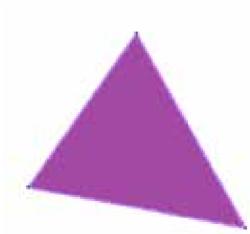
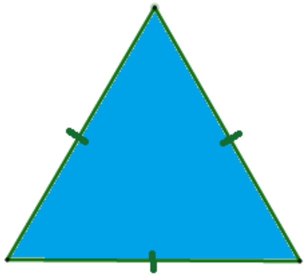
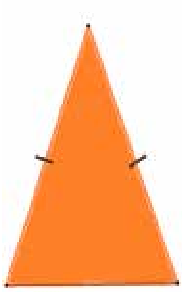
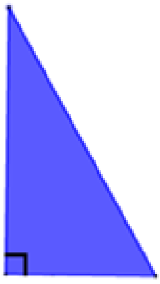
	$-7x+2y$	
Collection of terms (adding monomials)	$2\alpha^2\beta+10\alpha^2\beta-4\alpha^2\beta=$ $=(2+10-4)\alpha^2\beta=$ $=8\alpha^2\beta$	13
Adding polynomials	$(2x^2-5x+6)+(x^3+3x^2+4x-7)=$ $=2x^2-5x+6+x^3+3x^2+4x-7=$ $=x^3+5x^2-x-1$	13
Multiplying monomials	$3\alpha^2\beta\cdot5\alpha\beta^3\kappa=15\alpha^3\beta^4\kappa$	14
Multiplying polynomials with monomials	$-2x(3x^2-x+4)=-6x^3+2x^2-8x$	16
Factorisation (Common factor method)	$6x^4y^3+8x^3y=$ $=2x^2y\cdot3x^2y^2+2x^2y\cdot4=$ $=2x^2y\,(3x^2y^2+4)$	19-20
Multiplying a polynomial with another	$(2x-3)(5x-1)=10x^2-17x+3$	22-23
Factorisation (grouping terms method)	$\kappa\alpha+\lambda\alpha+\kappa\beta+\lambda\beta=(\alpha+\beta)(\kappa+\lambda)$	26-27
Developing the square sum of two terms	$(\alpha+\beta)^2=\alpha^2+2\alpha\beta+\beta^2$ $(\blacksquare+\bullet)^2=\blacksquare^2+2\blacksquare\bullet+\bullet^2$ e.g. $(3x+5)^2=9x^2+10x+25$ $(\alpha+\beta)^2=\alpha^2+2\alpha\beta+\beta^2$ $(3x+5)^2=(3x)^2+2\cdot3x\cdot5+5^2=9x^2+30x+25$	30 and 34
Developing the square difference of two terms	$(\alpha-\beta)^2=\alpha^2-2\alpha\beta+\beta^2$ $(\blacksquare-\bullet)^2=\blacksquare^2-2\blacksquare\bullet+\bullet^2$ e.g. $(3x-5)^2=9x^2-10x+25$ $(\alpha-\beta)^2=\alpha^2-2\alpha\beta+\beta^2$ $(3x-5)^2=(3x)^2-2\cdot3x\cdot5+5^2=9x^2-30x+25$	31 and 34
factorisation (developing squares)	$(\alpha+\beta)^2=\alpha^2+2\alpha\beta+\beta^2$ $\blacksquare^2+2\blacksquare\bullet+\bullet^2=(\blacksquare+\bullet)^2$ or $(\alpha-\beta)^2=\alpha^2-2\alpha\beta+\beta^2$ $\blacksquare^2-2\blacksquare\bullet+\bullet^2=(\blacksquare-\bullet)^2$ e.g. $4x^2+12x+9=(2x+3)^2$ $\alpha^2+2\alpha\beta+\beta^2=(\alpha+\beta)^2$ $\underbrace{(2x)^2}_{4x^2}+\underbrace{2\cdot2x\cdot3}_{12x}+\underbrace{3^2}_{9}=(2x+3)^2$	32-33 and 34
Product of a sum of two terms and their difference	$(\alpha+\beta)(\alpha-\beta)=\alpha^2-\beta^2$ e.g. $(3x+5)(3x-5)=(3x)^2-5^2=9x^2-25$	38

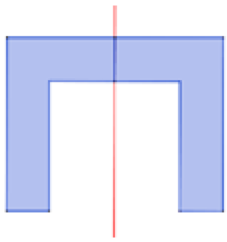
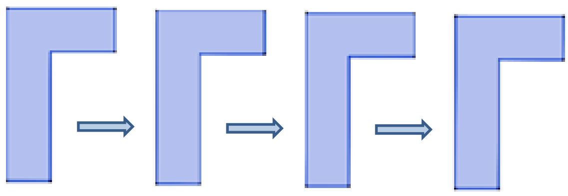
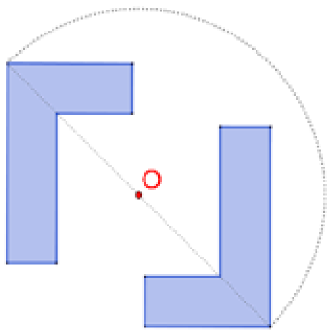
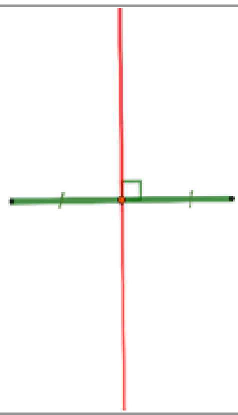
factorisation (square differences sum)	$\alpha^2 - \beta^2 = (\alpha + \beta)(\alpha - \beta)$ $e.g. 16x^2 - 25 = (4x - 5)(4x + 5)$ $\alpha^2 - \beta^2 = (\alpha - \beta)(\alpha + \beta)$ $16x^2 - 25 = (4x)^2 - 5^2 = (4x - 5)(4x + 5)$	39-40

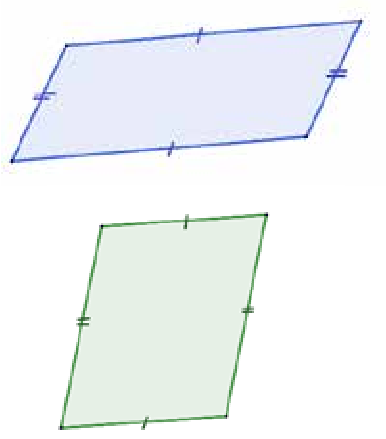
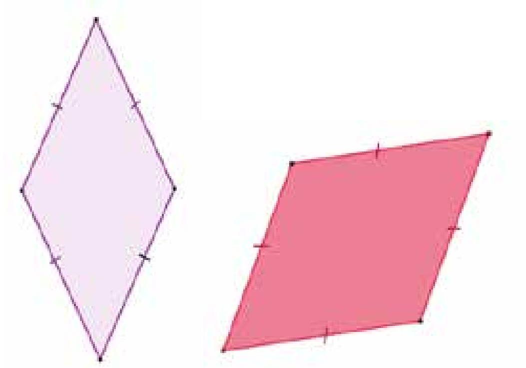
Geometry: Basic geometry notions		
Concept	**Example**	**Page**
Point A		
Line segment AB		
Straight line ε		
Halfline Aχ		
Angle $\hat{\omega}$		
Right angle		
Acute angle		
Obtuse angle		
Perpendicular lines		

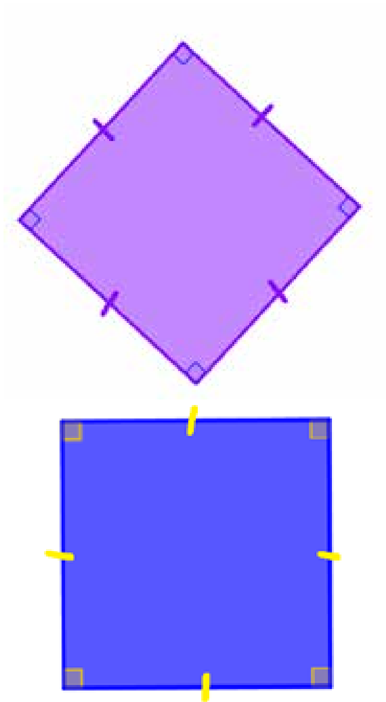
Circle		
Radius of a circle		
Diameter		
String		
Arc		
Tangent		

Protractor		
Central angle		
Intersecting lines		
Parallel lines		
Adjacent angles		
Vertically opposite angles		
Interior alternate angles		

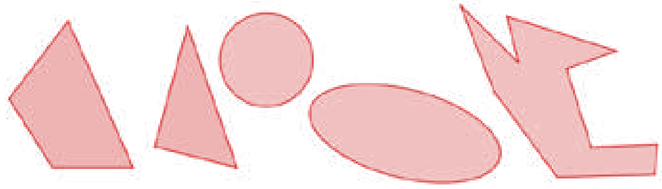
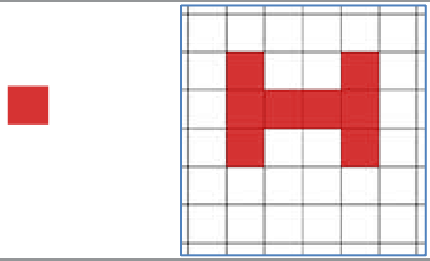
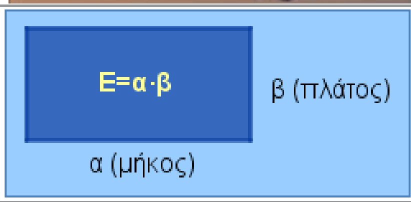
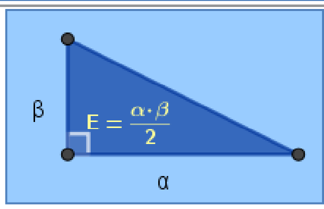
Corresponding angles		
Triangle		
Equilateral triangle		
Isosceles triangle		
Right triangle		
Obtuse-angled triangle		

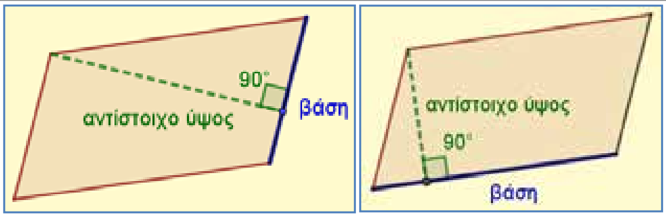
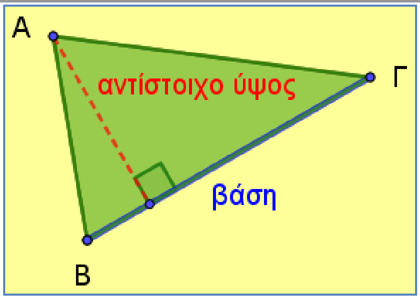
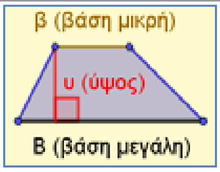
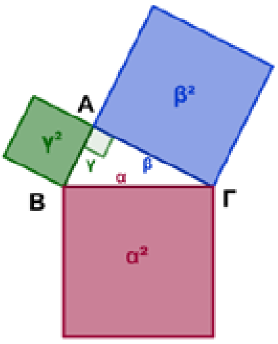
Geometry: Symmetry		
Concept	**Example**	**Page**
Line symmetry (reflection)		
Translation		
Rotational symmetry (180^{o} rotation)		
Perpendicular bisector to a line segment		

Parallelogram		
Rectangle		
Rhomboid (diamond)		

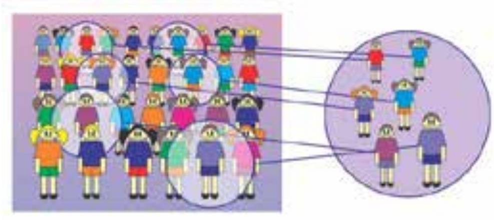
Square		

Glossary with examples

Area & Pythagorean theorem		
Concept	**Example**	**Page**
Surface		1
Area	A = 8 	4
Square centimetre		4
Square metre		5
Square milimetre		10
Area of a rectangle		5
Area of a right triangle		8

Area of a parallelogram	 A= (base) x (corresponding height)	14
Area of a triangle	 A = $\frac{(base)\,\mathbf{x}\,(corresponding\ height)}{\mathbf{2}}$	18
Area of a trapezoid	 A = $\frac{(\mathbf{B}+\mathrm{b})\cdot \mathrm{h}}{\mathbf{2}}$	20
Pythagorean theorem	 $\alpha^2=\beta^2+\gamma^2$	26

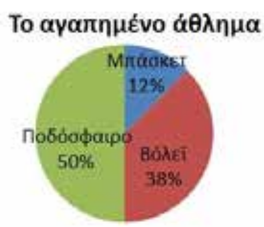
Glossary with examples

Stochastics: Statistics		
Concept	**Example**	
the population of a statistical study	Children aged 12 to 15. 	
sample		
a variable of a statistical study	What we investigate in our study. e.g. in a class with 16 children, • The number of mobile phones or tablets. • Favourite sport. • How long they talk on the phone in a day (in minutes).	
data	The observations or measurements we have for a variable. e.g., • 2, 1, 2, 1, 0, 1, 3, 1, 2, 1, 3, 0, 1, 2, 1, 0. • basketball, volleyball, football, volleyball, football, volleyball, basketball, volleyball, basketball, volleyball, football, volleyball, football, basketball, football, football. • 15, 45, 10, 43, 40, 15, 25, 30, 55, 20, 60, 35, 50, 35, 65, 40.	
the values of a variable	• 0, 1, 2, 3. • basketball, volleyball, football. • 10, 15, 20, 25, 30, 35, 40, 43, 45, 50, 55, 60, 65.	
the frequency of a value	How many times a value appears in the data. • The frequency of the value “3” = 2. • The frequency of the value “volleyball” = 6. • The frequency of the value “30” = 1.	
the relative frequency of a valye	$= \frac{frequency}{number\ of\ data}$. e.g. • The relative frequency of the value “3” = $\frac{2}{16}$. • The relative frequency of the value “volleyball” = $\frac{6}{16}$. • The relative frequency of the value “30” = $\frac{1}{16}$.	

<table>
<tr>
<td>grouping in classes</td>
<td>10, 15, 15, 20, 25, | 30, 35, 35, 40, 40, 43, 45, 50, 55, | 60, 65.
0≤time <30 — Class «0-30»
30≤time <60 — Class «30-60»
60≤time <90 — Class «60-90»</td>
<td></td>
</tr>
<tr>
<td>frequency table</td>
<td>
<table>
<tr><th>Number of mobile phones & tablets</th><th>Frequency</th></tr>
<tr><td>0</td><td>3</td></tr>
<tr><td>1</td><td>7</td></tr>
<tr><td>2</td><td>4</td></tr>
<tr><td>3</td><td>2</td></tr>
<tr><td>Total</td><td>16</td></tr>
</table>
</td>
<td></td>
</tr>
<tr>
<td>bar chart</td>
<td>


</td>
<td>Number of mobile phones or tablets per child

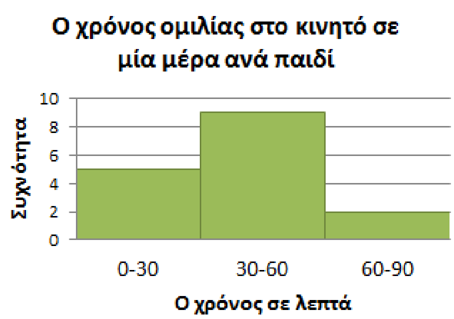
Frequency

Mobile phone or tablet</td>
</tr>
<tr>
<td>pie chart</td>
<td>


</td>
<td>My favourite sport

Football 50%

Basketball 12%

Volleyball 38%</td>
</tr>
<tr>
<td>histogram</td>
<td>


</td>
<td>Time spent talking on mobile phone every day per child

Frequency

Time (in minutes)</td>
</tr>
</table>

Funded by the Asylum, Migration and Integration Fund of the European Union

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